ICDEA 2016

The 22nd International Conference on Difference Equations and Applications

Osaka, Japan

July 24 to 29, 2016

Abstracts Book

Program of ICDEA2016

Sunday, July 24	
Time	**I-site Namba 2F**
16:00-17:00	**Registration**

Monday, July 25		
Time	**I-site Namba 2F**	
08:45-09:30	**Registration**	
	Conference Room 2 + Room 3	
	Chair	**Speaker and Title**
09:30-09:40	**Yamaoka**	**Opening**
09:40-10:40	**Cushing**	**Saber Elaydi (USA),** Global Dynamics of Difference Equations: Applications to Population Dynamics
10:40-11:00	**Coffee Break**	
11:00-12:00	**Schreiber**	**Ryusuke Kon (Japan),** Bifurcations in nonlinear Leslie matrix models
12:00-14:00	**Lunch**	
Session 1	**Conference Room 3**	
14:00-14:25	**Kon**	**Youssef Raffoul (USA),** Boundedness of Solutions in Almost Linear Volterra Difference Equations Using Fixed Point Theory and Lyapunov Functionals
14:25-14:50	**Kon**	**Yukihiko Nakata (Japan),** Stability of a logistic equation with multiple delays
14:50-15:15	**Raffoul**	**Kaori Saito (Japan),** On the Stability of an SIR epidemic discrete model with a delay
15:15-15:40	**Raffoul**	**Inese Bula (Latvia),** About neuron model with period two and three internal decay rate
15:40-16:00	**Coffee Break**	

Monday, July 25		
Session 1	Conference Room 3	
Time	Chair	Speaker and Title
16:00-16:25	Nakata	**Shinji Nakaoka (Japan),** Exploration of combinatorial emergence in coupled boolean network systems
16:25-16:50	Nakata	**Mansoor Saburov (Malaysia),** Applications of non-autonomous discrete dynamical systems into nonlinear consensus problems
16:50-17:15	Nakaoka	**Kuo-Chih Hung (Taiwan),** Allee effect and its applications
17:15-17:40	Nakaoka	**Wirot Tikjha (Thailand),** Prime period 4 behavior of certain piecewise linear system of difference equations where initial condition are some points in positive x-axis
18:30-20:30	**Welcome Party**	

Session 2	Conference Room 1	
Time	Chair	Speaker and Title
14:00-14:25	Nishimura	**Aiko Tanaka (Japan),** An evolutionary game model of families' voluntary provision of public goods
14:25-14:50	Nishimura	**Takashi Honda (Japan),** Operator theoretic phenomena of the Markov operators which are induced by stochastic difference equations
14:50-15:15	Nishimura	**Ryoji Hiraguchi (Japan),** On a solution path to the optimal growth model with multiplicative habits
15:15-15:40	Nishimura	**Takuma Kunieda (Japan),** Asset Bubbles, Financial Crisis, and Unemployment
15:40-16:00	**Coffee Break**	
16:00-16:25	D'Aniello	**Hikmet Kemaloglu (Turkey),** Uniqueness of Potential Function in Sturm-Liouville Difference Equation
16:25-16:50	D'Aniello	**Yan-Hsiou Cheng (Taiwan),** Eigenvalue estimates for the Sturm- Liouville equation
16:50-17:15	Kemaloglu	**Petr Zemánek (Czech Republic),** Self-adjoint extensions of minimal linear relation associated with discrete symplectic system
17:15-17:40	Kemaloglu	**Nobuyuki Higashimori (Japan),** Convergence of finite difference schemes applied to the Cauchy problems of quasi-linear partial differential equations of the normal form
18:30-20:30	**Welcome Party**	

Tuesday, July 26		
	Conference Room 2 + Room 3	
Time	**Chair**	**Speaker and Title**
09:00-10:00	Yu	**Christian Pötzsche (Austria),** Numerical dynamics of integrodifference equations
10:10-11:10	Zhang	**Eduardo Liz (Spain),** Complexity in discrete-time seasonal population models with harvesting
11:10-11:30	**Coffee Break**	
11:30-11:55	Liz	**István Győri (Hungary),** Asymptotic constancy in linear difference equations
12:00-14:00	**Lunch**	
Session 1	**Conference Room 3**	
14:00-14:25	Schreiber	**Ziyad Al-Sharawi (UAE),** A prey-predator model with refuge and toxic effect in prey
14:25-14:50	Schreiber	**Stephen Baigent (UK),** Convexity of invariant manifolds of Kolmogorov maps
14:50-15:15	Al-Sharawi	**Rafael Luis (Portugal),** Recent advances in global stability of monotone maps: application to population dynamics
15:15-15:40	Al-Sharawi	**Cónall Kelly (Jamaic),** Stabilisation of difference equations with noisy prediction-based control
15:40-16:00	**Coffee Break**	
16:00-16:25	Luis	**Roman Kozlov (Norway),** The adjoint equation method for constructing first integrals of delay differential equations
16:25-16:50	Luis	**Emma D'Aniello (Italy),** Iterated Function Systems and Attractors
16:50-17:15	Stehlík	**Jeremias Epperlein (Germany),** Cellular automata as topological dynamical systems and topological conjugacies between them
17:30-18:00	Pötzsche	**ISDE Meeting**

Tuesday, July 26		
Session 2	**Conference Room 1**	
Time	**Chair**	**Speaker and Title**
14:00-14:25	**Zdun**	**Elvan Akin (USA),** On Nonoscillatory Solutions of Two Dimensional Nonlinear Delay Time-Scale Systems
14:25-14:50	**Zdun**	**Agnieszka B. Malinowska (Poland),** The Cucker-Smale type models on isolated time scales
14:50-15:15	**Akin**	**Ewa Girejko (Poland),** Extension of the Cucker-Smale model to isolated time scales
15:15-15:40	**Akin**	**Andrejs Reinfelds (Latvia),** Integral stability for dynamic systems on time scales
15:40-16:00	**Coffee Break**	
16:00-16:25	**Bula**	**Marek Cezary Zdun (Poland),** Regular iterations connected with an initial value problem of a nonlinear difference equation
16:25-16:50	**Bula**	**Alexander Zuevsky (Czech Republic),** Non-commutative symmetries for generalized Davey-Stewartson equations
16:50-17:15	**Zhang**	**Linfeng Zhou (China),** Roughness of tempered exponential dichotomies

Wednesday, July 27		
	Conference Room 2 + Room 3	
Time	**Chair**	**Speaker and Title**
09:00-10:00	**Elaydi**	**Jim Cushing (USA),** Matrix Models Defined by Systems of Nonlinear Difference Equations and the Fundamental Bifurcation Theorem when the Projection Matrix is Imprimitive
10:10-11:10	**Kon**	**Sebastian Schreiber (USA),** Population Persistence and Species Coexistence in Random Environments
11:10-11:30	**Coffee Break**	
11:30-11:55	**Stehlík**	**Zuzana Došlá (Czech Republic),** Decaying solutions for discrete boundary value problems on the half line
12:00-	**Sightseeing Tour of Kyoto**	

Thursday, July 28

Time	Chair	Speaker and Title
	Conference Room 2 + Room 3	
09:00-10:00	Pötzsche	**Jianshe Yu (China),** Some problems on the global attractivity of linear nonautonomous difference equations
10:10-11:10	Liz	**Weinian Zhang (China),** Regularity of C^1 Linearization
11:10-11:30	**Coffee Break**	
11:30-11:55	Győri	**Mihály Pituk (Hungary),** The growth of positive solutions of difference equations with applications to delay differential equations
12:00-14:00	**Lunch**	
Session 1	**Conference Room 3**	
14:00-14:25	Došlá	**Jitsuro Sugie (Japan),** Nonoscillation theorems for second-order linear difference equations via the Riccati-type transformation
14:25-14:50	Došlá	**Jana Krejčová (Czech Republic),** Nonoscillatory solutions of the four-dimensional neutral difference system
14:50-15:15	Olaru	**Hideaki Izumi (Japan),** General terms of algebraic recurrence relations
15:15-15:40	Olaru	**Seiji Saito (Japan),** Converse theorems on globally asymptotic stability of solutions for nonlinear difference equations
15:40-16:00	**Coffee Break**	
16:00-16:25	Tikjha	**Ignacio Bajo (Spain),** Invariants for a class of discrete dynamical systems given by rational mappings
16:25-16:50	Tikjha	**Zachary A. Kudlak (USA),** A Second-Order Rational Difference Equation with a Quadratic Term
16:50-17:15	Bajo	**Aija Anisimova (Latvia),** Behaviour of Solutions of Some Rational Difference Equations with a Positive Real Power
18:30-20:30	**Conference Dinner**	

Session 2	Conference Room 1	
14:00-14:25	Malinowska	**Zbigniew Leśniak (Poland),** On topological invariants of Brouwer flows
14:25-14:50	Malinowska	**Dorota Mozyrska (Poland),** Fractional discrete-time of Hegselmann-Krause's type consensus model
14:50-15:15	Reinfelds	**Tatiana Odzijewicz (Poland),** Discrete fractional Cucker-Smale optimal control problem
15:15-15:40	Reinfelds	**Dzintra Šteinberga (Latvia),** Bounded solution of dynamic system on time scale
15:40-16:00	**Coffee Break**	

Thursday, July 28		
Session 2	**Conference Room 1**	
Time	**Chair**	**Speaker and Title**
16:00-16:25	**Hamaya**	**Yoshitsugu Kabeya (Japan),** Difference equation and the related Schrödinger operator
16:25-16:50	**Hamaya**	**Masanori Shiro (Japan),** Equations with five variables give the second Feigenbaum constant
16:50-17:15	**Kabeya**	**Toshiyuki Kohno (Japan),** On the behavior of the error in numerical iterative method for PDE
18:30-20:30	**Conference Dinner**	

Friday, July 29		
	Conference Room 2 + Room 3	
Time	**Chair**	**Speaker and Title**
09:00-10:00	**Győri**	**Petr Stehlík (Czech Republic),** Evolutionary games on graphs and discrete dynamical systems
Session 1	**Conference Room 3**	
10:10-10:35	**Pituk**	**Sorin Olaru (France),** Set Invariance for Delay Difference Equations
10:35-11:00	**Pituk**	**Yoshihiro Hamaya (Japan),** On the asymptotic stability of a discrete combat model
11:00-11:25	**Yamaoka**	**Masakazu Onitsuka (Japan),** On the Hyers-Ulam stability of a first-order linear difference equation
11:25-11:40	**Coffee Break**	
11:40-12:40	**Matsunaga**	**Hiroshi Kokubu (Japan),** Topological Computation Theory for the Global Dynamics of Multi-Parameter Systems

Session 2	**Conference Room 1**	
Time	**Chair**	**Speaker and Title**
10:10-10:35	**Cheng**	**Cheng-Hsiung Hsu (Taiwan),** Entire solutions in a bistable periodic lattice dynamical system
10:35-11:00	**Cheng**	**Jian Jhong Lin (Taiwan),** Traveling wave solutions for discrete-time model of delayed cellular neural networks
11:00-11:25	**Hsu**	**Tzi-Sheng Yang (Taiwan),** Stability for Monostable Wave Fronts of Delayed Lattice Differential Equations

Plenary Lectures

Matrix Models in Structured Population Dynamics and the Fundamental Bifurcation Theorem when the Projection Matrix is Imprimitive

JIM M. CUSHING

Department of Mathematics, University of Arizona
617 N Santa Rita, Tucson, AZ 85721, USA
cushing@math.arizona.edu

In population biology, the discrete time dynamics of structured populations are described by systems of difference equations, which when written in vector form are called matrix models. The vector of state variables x is iterated forward in time by the multiplication of a projection matrix $P(x)$. This matrix encodes the modeling assumptions about basic vital rates (birth, death, etc.) and state transitions (young to old, juvenile to adult, large to small, etc.) and how they depend on the state variable (through so-called density effects). The resulting nonlinear map $x(t+1) = P(x(t))x(t)$ has fixed point $x = 0$. Since extinction-versus-persistence is a basic concern in population biology, the stability of $x = 0$ is of fundamental importance. The linearization principle implies stability/instability of $x = 0$ can be determined by the eigenvalues of the Jacobian evaluated at $x = 0$, which is just $P(0)$. Generally P is primitive and, as a result, stability of $x = 0$ is determined by the (strictly) dominant eigenvalue r (the spectral radius) of $P(0)$. The nature of the bifurcation that occurs at $r = 1$ is, in this case, well understood: a global continuum of positive equilibria bifurcates from $x = 0$ at $r = 1$, which near the bifurcation point is stable if the bifurcation is forward and which is unstable if it is backwards. Backward bifurcations are of interest in one important way: they generally give rise to an interval of $r < 1$ values on which a strong Allee effect occurs and to a tipping point $r^* < 1$ below which a sudden population crash occurs (through a blue-sky bifurcation). There are, however, applications which give rise to matrix models with imprimitive projection matrices P. When P is imprimitive the bifurcation at $r = 1$ is more complicated and, in general, is not well understood. I will describe some example cases and some new theorems concerning the bifurcation at $r = 1$. These models and theorems are motivated by field observations of my ecologist collaborators working on Protection Island National Wildlife Refuge in the state of Washington. Our project concerns the effects of climate change on marine birds nesting on the Island and the prospect for their future survival in lieu of increasing mean sea surface temperature. Since adaptation is important in this regard, I will also describe an evolutionary version of matrix models and the form that the fundamental bifurcation theorem takes for the resulting "Darwinian" models.

Global Dynamics of Difference Equations: Applications to Population Dynamics

SABER ELAYDI

Trinity University
San Antonio, Texas, USA
selaydi@trinity.edu

In this talk we will present the latest development in the global dynamics of two types of systems generated by triangular maps and monotone maps. The dynamics of planar monotone maps have been well understood through the work of Hal Smith. The theory of monotone maps is now extended to higher dimensional maps via geometrical interpretation of monotonicity. Another class of maps for which the Global dynamics have been successfully established, is the class of triangular maps where the Jacobian matrix of the map is triangular. Applications of our theory to population biology will be presented. For instance, hierarchical models may be represented by triangular maps defined on R^k_+. In particular, we focus our attention on models with the Allee effect. The general theory of the global dynamics of triangular maps was established by Balreira, E., and Luis [1]. Here we extend these results to include the difficult case of non-hyperbolic maps, building upon the work by Assas et al. [2,3]. We show that in the case of non-hyperbolic maps, the center manifold is semi-stable from above. Finally, we show how immigration to one of the species or to both would change drastically the dynamics of the system. It is shown that if the level of immigration to one or both species is above a specified level, then there will be no extinction region.

References

[1] F. C. Balreira, S. Elaydi and R. Luis, *Global dynamics of triangular maps*, Nonlinear Anal. TMA Ser. A 104 (2014) 75–83.

[2] L. Assas, S. Elaydi, E. Kwessi, G. Livadiotis and D. Ribble, *Hierarchical competition models with the Allee effect*, J. Biol. Dyn. 9, Suppl. 1 (2015) 32–44.

[3] L. Assas, S. Elaydi, E. Kwessi, G. Livadiotis and B. Dennis, *Hierarchical competition models with the Allee effect II: the case of immigration*, J. Biol. Dyn. 9 (2015) 288–316.

Topological Computation Theory for the Global Dynamics of Multi-Parameter Systems

HIROSHI KOKUBU

Department of Mathematics, Kyoto University
Kyoto 606-8502, Japan
kokubu@math.kyoto-u.ac.jp

The fundamental theorem of dynamical systems due to C. Conley asserts that the compliment of the chain-recurrent set in the phase space is gradient-like for any finite-dimensional dynamical systems. In general, the chain-recurrent set, which is the weakest notion of recurrence in dynamics, has infinitely many component, and thus, it is hard to exactly capture it. Together with my collaborators, I have been working on developing a computational method, combined with several topological ideas, for obtaining a slightly weaker form of the "recurrent vs gradient-like decomposition" of the phase space, called the Morse decomposition of the global dynamics of multi-parameter systems. In this talk, I will explain the basic ideas of these topological computation methods, and discuss some applications and extensions.

References

[1] Zin Arai, William Kalies, Hiroshi Kokubu, Konstantin Mischaikow, Hiroe Oka and Paweł Pilarczyk, *A database schema for the analysis of global dynamics of multiparameter systems*, SIAM J. Appl. Dyn. Syst. 8 (2009) 757–789.

[2] Justin Bush, Marcio Gameiro, Shaun Harker, Hiroshi Kokubu, Konstantin Mischaikow, Ippei Obayashi and Pawel Pilarczyk, *Combinatorial-topological framework for the analysis of global dynamics*, Chaos 22 (2012) 047508.

[3] Hiroshi Kokubu and Hiroe Oka, *A topological computation approach to the interior crisis bifurcation*, Nonlinear Theory and its Applications IEICE 4 (2013) 97–103.

[4] Shaun Harker, Hiroshi Kokubu, Konstantin Mischaikow and Paweł Pilarczyk, *Inducing a map on homology from a correspondence*, Proc. Amer. Math. Soc. 144 (2016) 1787–1801.

Bifurcations in nonlinear Leslie matrix models

RYUSUKE KON

Faculty of Engineering, University of Miyazaki
Gakuen Kibanadai Nishi 1-1, Miyazaki 889-2192, Japan
konr@cc.miyazaki-u.ac.jp

This talk considers the dynamics of the following system of nonlinear difference equations

$$\begin{cases} u_{1,k+1} = f_1\phi_1(\mathbf{u}_k)u_{1,k} + f_2\phi_2(\mathbf{u}_k)u_{2,k} + \cdots + f_n\phi_n(\mathbf{u}_k)u_{n,k} \\ u_{2,k+1} = s_1\sigma_1(\mathbf{u}_k)u_{1,k} \\ u_{3,k+1} = s_2\sigma_2(\mathbf{u}_k)u_{2,k} \\ \qquad\vdots \\ u_{n,k+1} = s_{n-1}\sigma_{n-1}(\mathbf{u}_k)u_{n-1,k}, \qquad \mathbf{u}_k = (u_{1,k}, u_{2,k}, \dots, u_{n,k})^\top, \end{cases}$$

which is a nonlinear Leslie matrix model with n (≥ 2) age-classes. The variable $u_{i,k}$ denotes the number of individuals of age $i \in \{1, 2, \dots, n\}$ at time $k \in \{1, 2, \dots, \}$. Since the nonlinear Leslie matrix model exhibits complicated behavior even if $n = 2$ [1], we focus on the bifurcation of the population-free equilibrium point $\mathbf{u} = \mathbf{0}$ and show that the bifurcation problem can be seen as the stability problem for Lotka-Volterra differential equations if the population is semelparous (i.e., $f_1 = f_2 = \cdots = f_{n-1} = 0$). Furthermore, we extend this result to both iteroparous and multi-species cases. The study gives a mathematical base to [4] and rediscovers some results in [2, 3]

References

[1] J. Guckenheimer, G. Oster and A. Ipaktchi, *The dynamics of density dependent population models*, J. Math. Biol. 4 (1977) 101–147.

[2] J. M. Cushing, *Three stage semelparous Leslie models*, J. Math. Biol. 59 (2009) 75–104.

[3] J. M. Cushing and S. M. Henson, *Stable bifurcations in semelparous Leslie models*, J. Biol. Dyn. 6 (2012) 80–102.

[4] R. Kon, *Age-structured Lotka-Volterra equations for multiple semelparous populations*, SIAM J. Appl. Math. 71 (2011) 694–713.

Complexity in discrete-time seasonal population models with harvesting

EDUARDO LIZ

Departamento de Matemática Aplicada II, Universidad de Vigo
Campus Universitario, 36310 Vigo, Spain
eliz@dma.uvigo.es

Population dynamics of many species are influenced by seasonality, and seasonal interactions have the potential to modify important factors such as population abundance and population stability [4]. We consider a discrete semelparous population model with an annual cycle divided into a breeding and a non-breeding season, and introduce harvesting into the model following [1]. We report some interesting phenomena such as conditional and non-smooth hydra effects, coexistence of two nontrivial attractors, and hysteresis. Our results highlight the importance of several often underestimated issues that are crucial for management, such as census timing and intervention time.

Finally, we analyze the interaction of sequential density dependence with other factors that affect individual fitness: carry-over effects [2] and mating limitations leading to Allee effects [5].

References

[1] N. Jonzén and P. Lundberg, *Temporally structured density dependence and population management* Ann. Zool. Fennici 36 (1999) 39–44.

[2] E. Liz and A Ruiz-Herrera, *Potential impact of carry-over effects in the dynamics and management of seasonal populations*, PLoS One 11 (2016) e0155579.

[3] E. Liz, *Population responses to harvesting in a discrete model of sequential density dependence*, Preprint (2016).

[4] I. I. Ratikainen et al., *When density dependence is not instantaneous: theoretical developments and management implications*, Ecol. Lett. 11 (2008) 184–198.

[5] S. J. Schreiber, *Allee effects, extinctions, and chaotic transients in simple population models*, Theor. Popul. Biol. 64 (2003) 201–209.

Numerical dynamics of integrodifference equations

CHRISTIAN PÖTZSCHE

Institut für Mathematik, Alpen-Adria Universität Klagenfurt
Universitätsstraße 65–67, 9020 Klagenfurt, Austria
christian.poetzsche@aau.at

Integrodifference equations (IDEs for short) are a popular tool in theoretical ecology to describe the spatial dispersal of populations with nonoverlapping generations (cf. [1]).

From a mathematical perspective, IDEs are recursions on ambient spaces of continuous or integrable functions and therefore generate an infinite-dimensional dynamical system. Hence, for simulation purposes an appropriate numerical approximation yielding a finite-dimensional state space is due. Our goal is to study those dynamical properties of IDEs (e.g. existence of reference solutions, attractors, invariant manifolds) which are preserved under corresponding numerical methods and to establish convergence for increasingly more accurate schemes.

References

[1] M. Kot and W. Schaefer, *Discrete-time growth-dispersal models*, Math. Biosci. 80 (1986) 109–136.

[2] A. Stuart and A. Humphries, Dynamical Systems and Numerical Analysis, Monographs on Applied and Computational Mathematics, University Press, Cambridge, 1998.

Population Persistence and Species Coexistence in Random Environments

SEBASTIAN J. SCHREIBER

Department of Evolution and Ecology, University of California
Davis, CA 95616, USA
sschreiber@ucdavis.edu

Stochastic fluctuations in temperature, precipitation and a host of other environmental factors occur at multiple spatial and temporal scales. As the survival and reproduction of organisms, whether they be plants, animals, or viruses, depend on these environmental factors, these stochastic fluctuations often drive fluctuations in population abundances. This simple observation leads to a fundamental question in population biology. Namely, under what conditions do stochastic environmental fluctuations hinder or facilitate the maintenance of biodiversity? This question is particularly pressing in light of global climate models predicting increasing temporal variation in many climatic variables over the next century.

One fruitful approach to tackling this question from population biology is the development and analysis of models accounting for nonlinear feedbacks among species, population structure, and environmental stochasticity. In this talk, I will discuss progress in the development of a mathematical theory for stochastic coexistence where the dynamics of the interacting species are encoded by random difference equations and coexistence corresponds to the limit points of empirical measures being bounded away from an extinction set. I will illustrate the theory with empirical based examples involving checkerspot butterflies, Kansas prairies, and coastal dunes.

Evolutionary games on graphs and discrete dynamical systems

JEREMIAS EPPERLEIN, STEFAN SIEGMUND

Center for Dynamics & Institute for Analysis, Dept. of Mathematics
Technische Universität Dresden, 01062, Germany
jeremias.epperlein@tu-dresden.de, stefan.siegmund@tu-dresden.de

PETR STEHLÍK*

Dept. of Mathematics and NTIS, University of West Bohemia
Univerzitni 8, 30614 Pilsen, Czech Republic
pstehlik@kma.zcu.cz

Evolutionary games on graphs play an important role in the study of evolution of cooperation in applied biology. In this talk we discuss these discrete dynamical systems from the mathematical point of view in the deterministic case. We focus on coexistence equilibria, attractors, update rules and update orders. For example, we use constructive proofs to show that for all graphs there exist coexistence equilibria for certain game-theoretical parameters. Similarly, for all relevant game-theoretical parameters there exists a graph yielding coexistence equilibria. We conclude with a list of open problems.

References

[1] C. Hauert and M. Doebeli, *Spatial structure often inhibits the evolution of cooperation in the snowdrift game*, Nature 428 (2004) 643–646.

[2] M. A. Nowak, *Five rules for the evolution of cooperation*, Science 314 (2006) 1560–1563.

[3] J. Epperlein, S. Siegmund and P. Stehlík, *Evolutionary games on graphs and discrete dynamical systems*, J. Difference Equ. Appl. 21 (2015) 72–95.

[4] J. Epperlein, S. Siegmund, P. Stehlík and V. Švígler, *Coexistence equilibria of evolutionary games on graphs under deterministic imitation dynamics*, Discrete Contin. Dyn. Syst. Ser. B. 21 (2016) 803–813.

Some problems on the global attractivity of linear nonautonomous difference equations

JIANSHE YU

Guangzhou University
Guangzhou, 510006, P. R. China
jsyu@gzhu.net.cn

Main aim of this talk is to solve several problems on the global attractivity of the zero solution of the nonautonomous difference equation

$$x_{n+1} - x_n + P_n x_{n-k_n} = 0, \;\; n \in \mathbf{Z}(0),$$

where $\{P_n\}$ is a sequence of nonnegative real numbers, $\{k_n\}$ is a sequence of nonnegative integers with $n - k_n \to \infty$ as $n \to \infty$. Some open problems are posed on the global attractivity of equilibrium 1 for the discrete population model.

Regularity of C^1 Linearization

WEINIAN ZHANG

School of Mathematics, Sichuan University
Chengdu, Sichuan 610064, P. R. China
matzwn@126.com

C^1 linearization is of special interests because it can distinguish characteristic directions of dynamical systems. In this talk new advances on sharp regularity of C^1 linearization for planar hyperbolic diffeomorphisms are introduced. Moreover, results are given in the higher dimensional Euclinean space and Banach spaces in nonresonant cases and resonant cases. Those results are obtained jointly with Wenmeng Zhang and Witold Jarczyk.

References

[1] W. M. Zhang and Weinian Zhang, *C^1 linearization for planar contractions*, J. Funct. Anal. 260 (2011) 2043–2063.

[2] W. M. Zhang, Weinian Zhang and W. Jarczyk, *Sharp regularity of linearization for $C^{1,1}$ hyperbolic diffeomorphisms*, Math. Ann. 358 (2014) 69–113.

[3] W. M. Zhang and Weinian Zhang, *Sharpness for C^1 linearization of planar hyperbolic diffeomorphisms*, J. Differential Equations 257 (2014) 4470–4502.

[4] W. M. Zhang and Weinian Zhang, *α-Hölder linearization of hyperbolic diffeomorphisms with resonance*, Ergodic Theory Dynam. Systems 36 (2016) 310–334.

Contributed Talks

On Nonoscillatory Solutions of Two Dimensional Nonlinear Delay Time-Scale Systems

ELVAN AKIN*

Department of Mathematics and Statistics, Missouri S&T
Rolla, MO USA
akine@mst.edu

ÖZKAN ÖZTÜRK

Department of Mathematics and Statistics, Missouri S&T
Rolla, MO USA
oo976@mst.edu

In this talk, we consider nonlinear two dimensional systems of first order delay dynamic equations on time scales and obtain necessary and sufficient conditions to show the existence of nonoscillatory solutions. Our approach is based on the sign of components of solutions and we use Knaster and Schauder fixed point theorems. Examples will be given to illustrate some of our results as well.

References

[1] D. R. Anderson, *Oscillation and nonoscillation criteria for two-dimensional time-scale systems of first order nonlinear dynamic equations*, Electronic Journal of Differential Equations, Vol. 2009 (2009), No. 24, pp.1–13.

[2] M. Bohner and A. Peterson, Dynamic Equations on Time Scales: An Introduction with Applications, Birkhäuser, Boston, 2001.

[3] M. Bohner and A. Peterson, Advances in Dynamic Equations on Time Scales, Birkhäuser, Boston, 2003.

[4] Ö. Öztürk, E. Akın, *Nonoscillation criteria for two dimensional time scale systems*, Nonauton. Dyn. Syst. 2016: 3: 1–13.

[5] X. Zhang, *Nonoscillation criteria for nonlinear delay dynamic systems on time scales*, International Journal of Mathematical, Computational, Natural and Physical Engineering Vol:8, No:1, 2014.

A prey-predator model with refuge and toxic effect in prey

ZIYAD ALSHARAWI

Department of Mathematics and Statistics, American University of Sharjah
Sharjah, United Arab Emirates
zsharawi@aus.edu

(Collaborative work with Joydev Chattopadhyay, Indian Statistical Institute)

In this talk, we discuss the dynamics of a prey-predator model with both refuge and toxic effects of prey. In particular, we consider the discrete model

$$\begin{cases} x_{n+1} = \dfrac{krx_n}{k+(r-1)x_n} - \beta\alpha x_n y_n \\ y_{n+1} = (c\beta\alpha - \theta)x_n y_n - \mu y_n, \end{cases}$$

where r is the intrinsic growth rate of the toxic prey $(r > 1)$, k is the carrying capacity of the prey $(k > 0)$, β is the predation rate $(0 < \beta < 1)$, c is the conversion efficiency $(0 < c < 1)$, α is the portion of the prey population that does not take refuge $(0 \le \alpha \le 1)$, μ is the natural mortality rate of the predator population $(0 < \mu < 1)$ and θ is the toxic effect due to consumption of toxin producing prey $(0 \le \theta < 1)$.

We analyze stability of equilibrium solutions, and show that the predator free equilibrium is globally asymptotically stable if the prey density is below a threshold level. Boundedness, persistence and permanence of the system will be characterized. We show that the system undergoes Neimark-Sacker bifurcation. Also, we show that increasing the carrying capacity of the prey reduces the chance of coexistence of both populations. Thus large carrying capacity leads to the paradox of enrichment.

References

[1] Eduardo Gonzalez-Olivares and Rodrigo Ramos-Jiliberto, *Dynamic consequences of prey refuges in a simple model system: more prey, fewer predators and enhanced stability,* Ecological Modelling 166 (2003) 135–146.

[2] Y. A. Kuznetsov, Elements of Applied Bifurcation Theorey, Third Edition, Springer 2004.

[3] James N. McNair, *The effects of refuges on predator-prey interactions: A reconsideration,* Theor. Popul. Biol. 29 (1986) 38–63.

[4] M. L. Rosenzweig, *Paradox of enrichment: destabilization of exploitation ecosystems in ecological time,* Science 171 (1971) 385–387.

Behaviour of Solutions of Some Rational Difference Equations with a Positive Real Power

AIJA ANISIMOVA

Department of Mathematics, University of Latvia
Zeļļu iela 25, Rīga, LV-1002, Latvia
aija.anisimova@gmail.com

The author investigates the local and global stability character and the periodic nature of solutions of some second order rational difference equations with a positive real power in the form:

$$x_{n+1} = \frac{\alpha + \beta x_n^k + \gamma x_{n-1}^k}{A + B x_n^k + C x_{n-1}^k}, \quad n = 0, 1, 2, .., \tag{1}$$

with non-negative parameters $\alpha, \beta, \gamma, A, B, C$ and arbitrary non-negative initial conditions x_{-1}, x_0 such that the denominator is always positive, and $k \in (0, \infty)$.

The boundedness character of solutions of Eq.(1) have been studied in paper [1]; furthermore, there are given several open problems and conjectures about Eq.(1). In this talk the author would like to consider some of them.

References

[1] E. Camouzis, *Boundedness of solutions of a rational equation with a positive real power*, Int. J. Difference Equ. 8 (2013) 135–178.

[2] S. Jasarevic-Hrustic, Z. Nurkanovic, M. R. S. Kulenovic and E. Pilav, *Birkhoff normal forms, KAM theory and symmetries for certain second order rational difference equation with quadratic term*, Int. J. Difference Equ. 10 (2015) 181–199.

[3] M. R. S. Kulenovic and G. Ladas, Dynamics of Second Order Rational Difference Equations: With Open Problems and Conjectures, Chapman& Hall/CRC, Boca Raton, FL, 2008.

Convexity of invariant manifolds of Kolmogorov maps

STEPHEN BAIGENT

Department of Mathematics, UCL
London, WC1E 6BT UK
steve.baigent@ucl.ac.uk

I will show that the planar Leslie-Gower model has a globally attracting invariant manifold that is either convex, concave, or a line segement [1]. My apporach is based upon establishing when a given line segment is mapped to a convex or concave curve. I will also show the existence of convex and concave globally attracting invariant manifolds for the 3-species Leslie-Gower model. To do this I will first relate the Leslie-Gower model to a Kolmogorov differential equation. Convexity is then obtained by showing that all planes whose normals lie in a suitable invariant cone are mapped to convex surfaces [2].

References

[1] S. Baigent, *Convexity of the carrying simplex for discrete-time planar competitive Kolmogorov systems*, to appear in J. Difference Equ. Appl. DOI:10.1080/10236198.2015.1125895.

[2] S. Baigent, *Geometry of carrying simplices of 3-species competitive Lotka-Volterra systems*, Nonlinearity 26 (2013) 1001–1029.

Invariants for a class of discrete dynamical systems given by rational mappings

IGNACIO BAJO

Depto. Matemática Aplicada II, Universidad de Vigo
36310 Vigo, Spain
ibajo@dma.uvigo.es

An invariant or first integral of a discrete dynamical system $x(k+1) = F(x(k))$ with domain $\mathcal{D}$ is a non constant map $H : \mathcal{U} \subset \mathbb{K}^n \to \mathbb{K}$ defined in an open and dense subset $\mathcal{U}$ of $\mathcal{D}$ such that for all $x \in \mathcal{U}$ it holds $H(F(x)) = H(x)$.

In this work, we study the existence of invariants for a family of rational dynamical systems. Explicitly, let $\mathbb{K}$ denote either $\mathbb{R}$ or $\mathbb{C}$. We consider the discrete dynamical systems in an open domain $\mathcal{D}$ of $\mathbb{K}^n$ of the form

$$x(k+1) = F(x(k)) = (F_1(x(k)), \dots, F_n(x(k))), \quad x(k) \in \mathcal{D} \subset \mathbb{K}^n \tag{1}$$

where the functions $F_i : \mathcal{D} \subset \mathbb{K}^n \to \mathbb{K}$ are linear fractionals sharing denominator:

$$F_i(x) = \frac{a_{i1}x_1 + a_{i2}x_2 + \cdots + a_{in}x_n + c_i}{b_1x_1 + b_2x_2 + \cdots + b_nx_n + d}, \ \ i = 1, 2, \dots, n,$$

for $x = (x_1, x_2, \cdots, x_n)$ and all involved parameters in $\mathbb{K}$. Such systems can be written with the aid of homogeneous coordinates as the composition of a linear map in $\mathbb{K}^{n+1}$ with a certain projection and their behaviour is strongly determined by the spectral properties of the corresponding linear map.

We will prove that if $n \geq 2$ then every system of this kind admits an invariant, both in the real and in the complex case. More precisely, our main result will be

Theorem 1. *Consider $n > 1$. If the dynamical system given by (1) is defined in a nonempty open set $\mathcal{D}$, then it admits an invariant defined in an open and dense subset*

$$\mathcal{U} = \{x \in \mathcal{D} \ : \ \mathcal{Q}(x) \neq 0\},$$

where $\mathcal{Q}(x)$ is a polynomial of degree 2 defined by a couple of (not necessarily distinct) eigenvectors u_1, u_2 of a matrix defined by the coefficients of the components of F.

In fact, for a sufficiently large n several functionally independent invariants can be obtained and, in many cases, the invariant can be chosen as the quotient of two quadratic polynomials. In this cases one has, as a consequence, that every orbit of the system results to be contained in a certain F-invariant quadric.

References

[1] I. Bajo, *Invariants for certain discrete dynamical systems given by rational mappings*, Qual. Theory Dyn. Sys. (2016), DOI 10.1007/s12346-016-0201-4, 24 pp.

Stabilisation of difference equations with noisy prediction-based control

ELENA BRAVERMAN

Department of Mathematics, University of Calgary
Calgary, Alberta T2N1N4, Canada
maelena@math.ucalgary.ca

CÓNALL KELLY*, ALEXANDRA RODKINA

Department of Mathematics, The University of the West Indies
Mona, Kingston 7, Jamaica, W.I.
conall.kelly@uwimona.edu.jm; alexandra.rodkina@uwimona.edu.jm

Prediction-based control was introduced by Ushio and Yamamoto [2] as a method of stabilising unstable periodic orbits of difference equations. In this talk, we consider the influence of stochastic perturbations on stability of a unique positive equilibrium of a difference equation subject to prediction-based control. These stochastic perturbations may be multiplicative if they arise from stochastic variation of the control parameter, or additive if they reflect the presence of systemic noise. The class of equations under consideration includes common population models like the Ricker model, the truncated logistic model, and modifications of the Beverly-Holt equation.

We begin by relaxing the control parameter in the deterministic equation, and deriving a range of values for the parameter over which all solutions eventually enter an invariant interval. Then, by allowing the variation to be stochastic, we demonstrate sufficient conditions (less restrictive than known ones for the unperturbed equation) under which the positive equilibrium will be globally a.s. asymptotically stable: i.e. the presence of noise improves the known effectiveness of prediction-based control. Finally, we show that systemic noise has a "blurring" effect on the positive equilibrium, which can be made arbitrarily small by controlling the noise intensity.

The results presented in this talk will be illustrated by numerical examples, and are published in [1].

References

[1] E. Braverman, C. Kelly and A. Rodkina, *Stabilisation of difference equations with noisy prediction-based control*, Physica D 326 (2016) 21–31.

[2] T. Ushio and S. Yamamoto, *Prediction-based control of chaos*, Phys. Lett. A 264 (1999) 30–35.

About neuron model with period two and three internal decay rate

INESE BULA*

Faculty of Physics and Mathematics, University of Latvia
Zeļļu iela 25, Rīga LV-1002, Latvia
Institute of Mathematics and Computer Science, University of Latvia
Raiņa bulvāris 29, Rīga LV-1459, Latvia
ibula@lanet.lv

MICHAEL A. RADIN

Rochester Institute of Technology, School of Mathematical Sciences
Rochester, New York 14623, USA
michael.radin@rit.edu

In [1] a difference equation $x_{n+1} = \beta x_n - g(x_n),\ n = 0, 1, 2, \ldots,$ was analyzed as a single neuron model, where $\beta > 0$ is an internal decay rate and a signal function g is the following piecewise constant function with McCulloch-Pitts nonlinearity:

$$g(x) = \begin{cases} 1, & x \geq 0, \\ -1, & x < 0. \end{cases} \tag{1}$$

Now we will study the following non-autonomous piecewise linear difference equation:

$$x_{n+1} = \beta_n x_n - g(x_n), \quad n = 0, 1, 2, \ldots,$$

where $(\beta_n)_{n=0}^{\infty}$ is a period two or three sequence

$$\beta_n = \begin{cases} \beta_0, & \text{if } n = 2k, \\ \beta_1, & \text{if } n = 2k+1, \end{cases} \quad \text{or} \quad \beta_n = \begin{cases} \beta_0, & \text{if } n = 3k, \\ \beta_1, & \text{if } n = 3k+1, \\ \beta_2, & \text{if } n = 3k+2, \end{cases} \quad k = 0, 1, 2, \ldots$$

and g is in form (1). In [2] we have been studied this model where $(\beta_n)_{n=0}^{\infty}$ is a period two sequence. We will investigate the periodic behaviour and stability of solutions relative to the periodic internal decay rate.

References

[1] Z. Zhou, *Periodic orbits on discrete dynamical systems*, Comput. Math. Appl. 45 (2003) 1155–1161.

[2] I. Bula and M. A. Radin, *Periodic orbits of a neuron model with periodic internal decay rate*, Appl. Math. Comput. 266 (2015) 293–303.

Eigenvalue estimates for the Sturm-Liouville equation

YAN-HSIOU CHENG

Department of Mathematics and Information Education
National Taipei University of Education
Taipei 106, Taiwan
yhcheng@tea.ntue.edu.tw

Recently, the discrete Hamiltonian systems have wide attracted attention. An interesting model is the discrete Sturm-Liouville equation of the form

$$(J_n + Q_q)\, y = \lambda y$$

where Q_q is a diagonal matrix with diagonal elements $q_1, q_2, \ldots, q_n$ and J_n is the $n \times n$ tridiagonal matrix of the form

$$J_n = \begin{bmatrix} 2 & -1 & 0 & \cdots & 0 \\ -1 & 2 & -1 & \ddots & \vdots \\ 0 & -1 & 2 & \ddots & 0 \\ \vdots & \ddots & \ddots & \ddots & -1 \\ 0 & \cdots & 0 & -1 & 2 \end{bmatrix}.$$

From the theory of Jacobi matrices, we know that the eigenvalues of the discrete Sturm-Liouville operator $J_n + Q_q$ are all real and simple. In particular, most of studies are focus on the eigenvalue estimates, bound of eigenvalues and inverse spectral problem. The purpose of this talk is to introduce the problem of the gap of first two eigenvalues of the discrete Sturm-Liouville equation. We will find the optimal lower bound of the first eigenvalues gap and show the optimizer of the first eigenvalues gap is only when Q_q is a constant times the identity matrix.

References

[1] M. S. Ashbaugh and R. D. Benguria, *Some eigenvalue inequalities for a class of Jacobi matrices*, Linear Algebra Appl. 136 (1990) 215–234.

[2] Wei-Yu Hsu, *Some eigenvalue problems related to the discrete Sturm-Liouville operator*, M. S. thesis, National Tsing Hua University, Taiwan, 2012.

Iterated Function Systems and Attractors

EMMA D'ANIELLO

Dipartimento di Matematica e Fisica, Scuola Politecnica e delle Scienze di Base
Seconda Università degli Studi di Napoli, Viale Lincoln n.5, 81100 Caserta, ITALIA
emma.daniello@unina2.it

Let X be a compact metric space with $\mathcal{S} = \{S_1, \ldots, S_m\}$ a finite set of continuous maps from X to itself. Call a non-empty compact subset F of X an *attractor* (or *invariant set*) for the iterated function system (IFS) $\mathcal{S}$ if $F = \cup_{i=1}^{m} S_i(F)$. We investigate the structure (geometry, Hausdorff dimension, etc...) of the invariant sets according to the properties of the functions in the generating systems. In particular, we focus our attention on the case when $X = [0,1]^n$, $n \geq 1$.

References

[1] A. Blokh, A. M. Bruckner, P. D. Humke and J. Smítal, *The space of ω-limit sets of a continuous map of the interval*, Trans. Amer. Math. Soc. 348 (1996) 1357–1372.

[2] A. M. Bruckner and T. H. Steele, *The Lipschitz structure of continuous self-maps of generic compact sets*, J. Math. Anal. Appl. 188 (1994) 798–808.

[3] E. D'Aniello and T. H. Steele, *Attractors for iterated function systems*, J. Fractal Geom. 3 (2016) 95–117.

[4] E. D'Aniello and T. H. Steele, *Attractors for iterated function schemes on $[0,1]^N$ are exceptional*, J. Math. Anal. Appl. 424 (2015) 537–541.

[5] E. D'Aniello, P. D. Humke and T. H. Steele, *The space of adding machines generated by continuous self maps of manifolds*, Topology Appl. 157 (2010) 954–960.

[6] E. D'Aniello, U. B. Darji and T. H. Steele, *Ubiquity of odometers in topological dynamical systems*, Topology Appl. 156 (2008) 240–245.

[7] E. D'Aniello and T. H. Steele, *A C^1 function for which the ω-limit points are not contained in the closure of the periodic points*, Trans. Amer. Math. Soc. 355 (2003) 2545–2556.

[8] Feng De-Jun and Wang Yang, *On the structures of generating iterated function systems of Cantor sets*, Adv. Math. 222 (2009) 1964–1981.

[9] K. Falconer, Fractal Geometry: Mathematical Foundations and Applications, John Wiley and Sons, 1990.

[10] J. E. Hutchinson, *Fractals and self-similarity*, Indiana Univ. Math. J. 30 (1981) 713–747.

Decaying solutions for discrete boundary value problems on the half line

ZUZANA DOŠLÁ*

Department of Mathematics and Statistics, Masaryk University
Kotlářská 2, CZ-61137 Brno, Czech Republic
dosla@math.muni.cz

MAURO MARINI , SERENA MATUCCI

Department of Mathematics and Informatics “Ulisse Dini”, University of Florence
I-50139 Florence, Italy
mauro.marini@unifi.it; serena.matucci@unifi.it

Consider the functional difference equation

$$\Delta(a_n|\Delta x_n|^{\alpha}\text{sgn}\,(\Delta x_n)) = \lambda F(n, x_{n+q}), \tag{1}$$

where $\lambda > 0$ is a real parameter, $\alpha > 0$, $q \in \{0, 1, 2\}$.

Some nonlocal boundary value problems, associated to (1) on unbounded domains, are studied by means of a new approach. Their solvability is obtained by using properties of the recessive solution to suitable half-linear difference equations, a half-linearization technique and a fixed point theorem in Fréchet spaces. The result is applied to derive the existence of nonoscillatory solutions with initial and final data.

References

[1] M. Cecchi, Z. Došlá, M. Marini and I. Vrkoč, *Summation inequalities and half-linear difference equations*, J. Math. Anal. Appl. 302 (2005) 1–13.

[2] Z. Došlá, M. Marini and S. Matucci, *Decaying solutions for discrete boundary value problems on the half line*, to appear in J. Difference Equ. Appl (2016).

[3] M. Marini, S. Matucci and P. Řehák, *Boundary value problems for functional difference equations on infinite intervals*, Adv. Difference Equ. 2006, Art. 31283, 14pp.

Cellular automata as topological dynamical systems and topological conjugacies between them

JEREMIAS EPPERLEIN

Institute for Analysis & Center for Dynamics
Department of Mathematics, TU Dresden, Germany
jeremias.epperlein@tu-dresden.de

Cellular automata are discrete dynamical systems that are used to model a wide variety of phenomena in biology, computer science, traffic simulation, etc. They are are also of interest from a purely mathematical point of view.

Since CA can be characterized as the continuous shift-commuting self-maps of subshifts, topological dynamics provides a natural framework for their investigation. Starting with the fundamental work [1] of Hedlund, this line of research proved to be very fruitful yielding nice results and even more intriguing open problems (see for example [2]).

We will review some of these questions and results and present some new ones concerning topological conjugacies between cellular automata.

References

[1] G. Hedlund, *Endomorphisms and automorphisms of the shift dynamical system.*, Mathematical Systems Theory 3 (1969) 320–375.

[2] P. Kurka, *Topological dynamics of one-dimensional cellular automata*, Encyclopedia of Complexity and System Sciences (R. A. Meyers, ed.) Part 20, 9246–9268, Springer-Verlag 2009.

The Cucker-Smale type models on isolated time scales

EWA GIREJKO

Faculty of Computer Science, Bialystok University of Technology
15-351 Białystok, Poland
e.girejko@pb.edu.pl

LUÍS MACHADO

Department of Mathematics, UTAD, 5001-801 Vila Real, and Institute of Systems and Robotics, University of Coimbra - Polo II, 3030-290 Coimbra, Portugal
lmiguel@utad.pt

AGNIESZKA B. MALINOWSKA*

Faculty of Computer Science, Bialystok University of Technology
15-351 Białystok, Poland
a.malinowska@pb.edu.pl

NATÁLIA MARTINS

Center for Research and Development in Mathematics and Applications
Department of Mathematics, University of Aveiro
3810-193 Aveiro, Portugal
natalia@ua.pt

We shall present results concerning two special cases of the following model:

$$\begin{cases} x_i^{\Delta}(t) = v_i(t) \\ v_i^{\Delta}(t) = Av(t) \end{cases} \tag{1}$$

for $i = 1, ..., N$, where $t \in \mathbb{T}$ being an isolated time scale. The evolution of agent i is described by $t \mapsto (x_i(t), v_i(t)) \in \mathbb{E}^2$, where $\mathbb{E}$ a finite dimensional inner product real space. For each t, $x_i(t)$ represents the *state* and $v_i(t)$ its *consensus parameter* at time t. Matrix $A = (a_{ij})_{i,j=1}^{N}$ with $a_{ij} \in \mathbb{R}$, quantifies the way the agents influence each other, f^{Δ} denotes the $\Delta-$derivative of f. It is of interest to know whether the system will converge to a consensus pattern, characterized by the fact that all the consensus parameter tend to a common value.

This work is supported by the Polish National Science Center grant on the basis of decision DEC-2014/15/B/ST7/05270.

Extension of the Cucker-Smale model to isolated time scales

EWA GIREJKO*

Bialystok University of Technology
ul. Wiejska 45A, Białystok, Poland
e.girejko@pb.edu.pl

AGNIESZKA B. MALINOWSKA

Bialystok University of Technology
ul. Wiejska 45A, Białystok, Poland
a.malinowska@pb.edu.pl

EWA SCHMEIDEL

Institute of Mathematics, University of Bialystok
Bialystok, Poland
eschmeidel@math.uwb.edu.pl

MAŁGORZATA ZDANOWICZ

Institute of Mathematics, University of Bialystok
Bialystok, Poland
mzdan@math.uwb.edu.pl

The purpose of this talk is to present the Cucker-Smale model on isolated time scales. This dynamical system models a consensus of emergence in a population of autonomous agents. The results establishing conditions under which such consensus occurs will be presented. We will present analytical methods as well as computer simulations.

The work was supported by Polish founds of National Science Center, granted on the basis of decision DEC-2014/15/B/ST7/05270.

Asymptotic constancy in linear difference equations

ISTVÁN GYŐRI

Department of Mathematics, University of Pannonia
H-8200 Veszprém, Hungary
gyori@almos.uni-pannon.hu

It is found that every solution of a system of linear delay difference equations has finite limit at infinity if some conditions are satisfied. These conditions are much weaker than the known sufficient conditions for asymptotic constancy of the solutions. When we impose some positivity assumptions on the coefficient matrices, our conditions are also necessary. The novelty of our results is illustrated by examples. This is a joint work with László Horváth (University of Pannonia, Hungary).

References

[1] I. Győri and L. Horváth, *Asymptotic constancy in linear difference equations: Limit formulae and sharp conditions*, Adv. Difference Equ. 2010, Art. ID 789302, 20 pp.

On the asymptotic stability of a discrete combat model

YOSHIHIRO HAMAYA

Department of Information Science, Okayama University of Science
1-1 Ridaichyo, Kitaku, Okayama 700-0005, Japan
hamaya@mis.ous.ac.jp

A model for the attrition of combat power is treated with some mathematical discrete combat model. In this talk, we consider a sufficient condition for the asymptotic stability of a discrete combat model, which appears as a model for a government army vs an anti-government army of guerrilla, a government army vs a terror army of guerrilla and an anti-government army of guerrilla vs a terror army of guerrilla, respectively, by applying the technique of a luxury Liapunov functional.

References

[1] M. J. Artelli and R. F. Deckro, *Modeling the Lanchester laws with system dynamics*, J. Defense Modeling and Simulation 5 (2008) 1–20.

[2] S. Elaydi, An Introduction to Difference Equations, 3rd ed., Springer, 2005.

[3] Y. Hamaya and K. Saito, *Global asymptotic stability of a delayed SIR epidemic model with diffusion*, to appear.

[4] D. R. Howes and R. M. Thrall, *A theory of ideal linear weights for heterogeneous combat forces*, Naval Research Logistics Qurtery 20 (1973) 645–659.

[5] F. W. Lanchester, *Aircraft in Warfare; The Dawn of the Fourth Arm*, 1916.

Asset Bubbles, Financial Crisis, and Unemployment

KEN-ICHI HASHIMOTO

Graduate School of Economics, Kobe University
Kobe 657-8501, Japan
hashimoto@econ.kobe-u.ac.jp

RYONFUN IM

Graduate School of Economics, Kobe University
Kobe 657-8501, Japan
tdrgsk@gmail.com

TAKUMA KUNIEDA*

School of Economics, Kwansei Gakuin University
Nishinomiya 662-8501, Japan
tkunieda@kwansei.ac.jp

A tractable overlapping-generations model with asset bubbles is presented to demonstrate that a financial crisis with the bursting of asset bubbles decreases economic growth rates and increases unemployment rates. In our model, without asset bubbles, all agents engage in capital production regardless of their idiosyncratic productivity shocks. A bubbly asset, which is intrinsically useless, has a positive market value because selling the asset is a fund-raising method for agents who draw sufficiently high productivity to initiate an investment project and purchasing the asset is a sole saving method for agents who draw too low productivity. The presence of asset bubbles corrects allocative inefficiency, relocating investment resources from low productive agents to high productive agents. Accordingly, the presence of asset bubbles promotes economic growth and reduces unemployment rates. However, extrinsic uncertainty bursts asset bubbles and causes a self-fulfilling financial crisis. High unemployment rates follow a self-fulfilling financial crisis.

References

[1] A. Martin and J. Ventura, *Economic growth with bubbles*, American Economic Review 102 (2012) 3033–3058.

Convergence of finite difference schemes applied to the Cauchy problems of quasi-linear partial differential equations of the normal form

NOBUYUKI HIGASHIMORI*

Center for the Promotion of Interdisciplinary Education and Research, Kyoto University
Yoshida-honmachi, Sakyo-ku, Kyoto 606-8501, Japan
nobuyuki@acs.i.kyoto-u.ac.jp

HIROSHI FUJIWARA, YUUSUKE ISO

Graduate School of Informatics, Kyoto University
Yoshida-honmachi, Sakyo-ku, Kyoto 606-8501, Japan
fujiwara@acs.i.kyoto-u.ac.jp; iso@acs.i.kyoto-u.ac.jp

We show a result on convergence of a consistent finite difference scheme to solve the Cauchy problem, including the ill-posed cases, of the following form:

$$\frac{\partial u_i}{\partial t} = \sum_{j=1}^{m}\sum_{l=1}^{n} a_{ijl}(t,x,u)\frac{\partial u_j}{\partial x_l} + f_i(t,x,u), \;\; u_i(0,x) = g_i(x) \quad (i = 1,\ldots,m). \tag{1}$$

We suppose analyticity with respect to $x = (x_1, \ldots, x_n)$ and $u = (u_1, \ldots, u_m)$ of the data (a_{ijl}, f_i, g_i) and merely continuity with respect to t, as in the Nirenberg-Nishida theorem. For the linear case, we already have a similar result [1]. Our proof is based on a discrete version of the proof given in [2], and our result is realized numerically on multiple-precision arithmetic environments, such as *exflib* [3].

References

[1] Y. Iso, *Convergence of a semi-discrete finite difference scheme applied to the abstract Cauchy problem on a scale of Banach spaces*, Proc. Japan Acad. Ser. A 87 (2011) 109–113.

[2] L. Nirenberg, *An abstract form of the nonlinear Cauchy-Kowalewski theorem*, J. Differential Geom. 6 (1972) 561–576.

[3] H. Fujiwara, http://www-an.acs.i.kyoto-u.ac.jp/~fujiwara/exflib

On a solution path to the optimal growth model with multiplicative habits

RYOJI HIRAGUCHI

Faculty of Law and Economics, Chiba University
Yayoicho 1-33, Inage, Chiba 263-8522, Japan
ryojih@gmail.com

In this paper, we study the optimal growth model in which consumption habits enter the utility function multiplicatively, and characterize the necessary and sufficient conditions that the optimal path must satisfy. The utility function with multiplicative habits is not concave and most of the existing theorems on the optimal control problems cannot be directly applicable to the model. To obtain the equilibrium conditions, we re-express the optimization problem in terms of the logarithm of the consumption and that of the habit stock so that the new optimization problem is concave and the Lagrangian is available. We then use these conditions to prove that the steady state growth path always exists. We also study the stability of the optimal growth path out of the steady state.

References

[1] C. Carroll, *Solving consumption models with multiplicative habits*, Economics Letters 68 (2000) 67–77.

[2] R. Hiraguchi, *Some foundations for multiplicative habits models*, Journal of Macroeconomics 30 (2008) 873–884.

Operator theoretic phenomena of the Markov operators which are induced by stochastic difference equations

TAKASHI HONDA*

Department of Mathematics, Faculty of Education, Iwate University
Morioka 020-8550, Japan
thonda7@iwate-u.ac.jp

YUKIKO IWATA

Meteorological College
Chiba 277-0852, Japan
yiwata@mc-jma.go.jp

The purpose of this talk is to present the relation between the Jacobs-de Leeuw-Glicksberg decomposition of semigroups and the spectral decomposition of the Markov operators which are induced by stochastic difference equations. Recently we proved new results about both decomposition. In this talk, we shall show the relation between both results.

References

[1] T. Honda, *A nonlinear analytic approach to the splitting theorem of Jacobs - de Leeuw - Glicksberg*, Nonlinear Anal. 75 (2012) 6002–6008.

[2] Y. Iwata, *Constrictive Markov operators induced by Markov processes*, Positivity 20 (2016) 355–367.

Traveling wave solutions for discrete-time model of delayed cellular neural networks

CHENG-HSIUNG HSU

Department of Mathematics, National Central University
Chung-Li 32001, Taiwan
chhsu@math.ncu.edu.tw

JIAN-JHONG LIN*

General Education Center, National Taipei University of Technology
Taipei 10608, Taiwan
jjlin@ntut.edu.tw

In this talk, we are concerned with the existence and stability of traveling wave solutions for the following partial difference equation

$$u_i(n+1) = (1-h)u_i(n) + h\sum_{l=0}^{d} \alpha_l f(u_{i-l}(n-\kappa_l)) + h\sum_{l=0}^{d} \beta_l f(u_{i-l}(n-\hat{\kappa}_l)), \quad (1)$$

where $i, n \in \mathbb{Z}$, $h > 0$, $\alpha_l, \beta_l \in \mathbb{R}$, $d, \kappa_l, \hat{\kappa}_l$ are nonnegative integer and f is defined by

$$f(x) = \frac{1}{2}(|x+1| - |x-1|).$$

This partial difference equation can be regarded as a discrete-time model of delayed cellular neural networks. Using the method of step along with positive characteristic roots of the equations, we successfully prove the existence of traveling wave solutions. Moreover, we also show that all such solutions are unstable. Additionally, we provide some numerical results to support our results, and point out the different structures of traveling wave solutions between the continuous-time and discrete-time models.

References

[1] L. O. Chau and L. Yang, *Cellular neural networks: Theory*, IEEE Trans. Circuits Syst. 35 (1988) 1257–1272.

[2] C.-H. Hsu and S.-S. Lin, *Existence and multiplicity of traveling waves in a lattice dynamical system*, J. Differential Equations 164 (2000) 431–450.

[3] C.-H. Hsu and S.-Y. Yang, *On camel-like traveling wave solutions in cellular neural networks*, J. Differential Equations 196 (2004) 481–514.

Entire solutions in a bistable periodic lattice dynamical system

CHENG-HSIUNG HSU*

Department of Mathematical, National Central University
Chungli 32001, Taiwan
chhsu@math.ncu.edu.tw

SHI-LIANG WU

School of Mathematics and Statistics, Xidian University
Xi'an, Shaanxi 710071, China
slwu@xidian.edu.cn

We are interested in finding entire solutions of a bistable periodic lattice dynamical system. By constructing appropriate super- and subsolutions of the system, we establish two different types of merging-front entire solutions. The first type can be characterized by two monostable fronts merging and converging to a single bistable front; while the second type is a solution which behaves as a monostable front merging with a bistable front and one chases another from the same side of x-axis.

References

[1] X. Chen, J.-S. Guo and C. C. Wu, *Traveling waves in discrete periodic media for bistable dynamics*, Arch. Rational Mech. Anal. 189 (2008) 189–236.

[2] J.-S. Guo and F. Hamel, *Front propagation for discrete periodic monostable equations*, Math. Ann. 335 (2006) 489–525.

[3] J.-S. Guo and Y. Morita, *Entire solutions of reaction-diffusion equations and an application to discrete diffusive equations*, Discrete Contin. Dyn. Syst. 12 (2005) 193–212.

[4] X. Liang and X. Zhao, *Spreading speeds and traveling waves for abstract monostable evolution systems*, J. Funct. Anal. 259 (2010) 857–903.

[5] N.W. Liu, W.T. Li and Z.C. Wang, *Entire solutions of reaction-advection-diffusion equations with bistable nonlinearity in cylinders*, J. Differential Equations 246 (2009) 4249–4267.

[6] Y. Morita and H. Ninomiya, *Entire solutions with merging fronts to reaction-diffusion equations*, J. Dynam. Differential Equations 18 (2006) 841–861.

[7] S.-L. Wu, Z.-X. Shi and F.-Y. Yang, *Entire solutions in periodic lattice dynamical systems*, J. Differential Equations 255 (2013) 3505–3535.

Allee effect and its applications

KUO-CHIH HUNG

Fundamental General Education Center, National Chin-Yi University of Technology
Taichung 41170, Taiwan
kchung@ncut.edu.tw

In theoretical ecology, a well known difference equation describing closed single species populations that reproduce synchronously at discrete time intervals is the following equation

$$x(t+1) = x(t)g(x(t)), \tag{1}$$

where $x(t)$ is the population size at generation t. The per capita growth rate $g(x(t))$ is of Allee effect type (that is, a decrease in population growth at low population sizes). We study the Allee effect in generating multiple attractors. We also give an application to a diffusive logistic equation with predation of Holling type II functional response.

References

[1] F. Courchamp, L. Berec and J. Gascoigne, Allee Effects in Ecologyand Conservation, Oxford University Press, 2008.

[2] I. Scheuring, *Allee effect increases the dynamical stability of populations*, J. Theor. Biol. 199 (1999) 407–414.

[3] M. R. S. Kulenovic and A.-A. Yakubu, *Compensatory versus overcompensatory dynamics in density-dependent Leslie models*, J. Difference Equ. Appl. 10 (2004) 1251–1265.

General terms of algebraic recurrence relations

HIDEAKI IZUMI*

Mathematics Division, Chiba Institute of Technology
Narashino 275-0023, Japan
izumi.hideaki@it-chiba.ac.jp

MAKOTO TSUKADA

Department of Computer Sciences, Toho University
Funabashi 274-8510, Japan
tsukada@is.sci.toho-u.ac.jp

SHIN-EI TAKAHASHI

Professor emeritus, Yamagata University
Yonezawa 992-8510, Japan
sin_ei1@yahoo.co.jp

In this talk, for a given algebraic recurrence relation

$$P(a_n,\ a_{n+1}, \ldots,\ a_{n+k-1}) = 0, \tag{1}$$

on a real numerical sequence $\{a_n\}$, where $P(x_0, \ldots,\ x_{k-1})$ is a polynomial of k variables, we present algorithms to obtain general terms of $\{a_n\}$ as formal series. As an application, we investigate a recurrence relation

$$a_{n+2} = \frac{a_n}{1 + a_{n+1}},\ a_0 > 0,\ a_1 > 0, \tag{2}$$

and determine the set $D = \{(a_0,\ a_1) \in \mathbb{R}_+^2 \mid \lim_{n\to\infty} a_n = 0\}$, which is proven non-empty in [1].

References

[1] S. Stević, *On the recursive sequence* $x_{n+1} = x_{n-1}/g(x_n)$, Taiwanese J. Math. 6 (2002) 405–414.

Difference equation and the related Schrödinger operator

YOSHITSUGU KABEYA

Department of Mathematical Sciences, Osaka Prefecture University
Sakai 599-8531, Japan
kabeya@ms.osakafu-u.ac.jp

This talk is based on the joint work with Professors Kazuhiro Ishige of Tohoku University and El Maati Ouhabaz of Université de Bordeaux.

We consider the difference equation of the form

$$\frac{u_{n+1}+u_{n-1}-2u_n}{h^2}+\frac{(N-1)(u_{n+1}-u_n)}{nh^2}+V(nh)u_n=0,$$

where $N \geq 2$ is a fixed natural number, $n \in \mathbb{N}$, $h > 0$, and V is a given function. For suitably given u_0 and u_1, we discuss the behavior of u_n as $n \to \infty$. Also, as a limit $h \to 0$, we discuss properties of radial solutions to $-\Delta u + V(|x|)u = 0$.

References

[1] K. Ishige and Y. Kabeya, *L^p norms of nonnegative Schrödinger heat semigroup and the large time behavior of hot spots*, J. Funct. Anal. 262 (2012) 2695–2733.

[2] M. Murata, *Structure of positive solutions to $(-\Delta + V)u = 0$ in R^n*, Duke Math. J. 53 (1986) 869–943.

Uniqueness of Potential Function in Sturm-Liouville Difference Equation

HIKMET KEMALOGLU(KOYUNBAKAN)

Department of Mathematic, Firat University
Elazig-23100, Turkey
hkoyunbakan@gmail.com

The aim of this talk is to present some uniqueness theorems for discrete Sturm-Liouville equation

$$-\Delta^2 y(n) + q(n)y(n+1) = \lambda y(n+1),\ n \in Z, \tag{1}$$

and Dirichlet boundary condition

$$y(0) = y(N+1) = 0 \tag{2}$$

by using the spectrum (collection of eigenvalues), where the sequence $q = [q(n)]_{n\in Z}$ is referred to as the potential. As usual, Δ is the forward difference operator defined by

$$\Delta y(n) = y(n+1) - y(n),\ \Delta^2 y(n) = y(n+2) - 2y(n+1) + y(n).$$

It is well known that the problem (1), (2) has N simple real eigenvalues with corresponding orthogonal eigenfunctions. By using the weighted numbers and eigenvalues, we will prove some uniqueness theorems analogue of continuous Sturm-Liouville problems.

References

[1] O. H. Hald, *Discrete inverse Sturm-Liouville problems. I. Uniqueness for Symmetric potentials*, Numer. Math. 27 (1976/77) 249–256.

[2] W. Kelley and A. Peterson, Difference Equations: An Introduction with Applications, 2nd ed., Harcourt/Academic Press, San Diego, 2001.

[3] M. Bohner and H. Koyunbakan, *Inverse Sturm-Liouville difference equations*, Filomat (2016).

On the behavior of the error in numerical iterative method for PDE

TOSHIYUKI KOHNO

Department of Mathematical Information Science, Okayama University of Science
Okayama 700-0005, Japan
kohno@mis.ous.ac.jp

The purpose of this talk is to present the behabior of the error in classical iterative method, Jacobi, Gauss-Siedel and SOR, for solving a partial differential equations. The coefficient matrix given by using difference approximation is very simple. This problem is a just right problem to check the property of the classical method.
We consider the partial differential equation.

$$\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial x^2} = f(x, y), \quad x, y \in \Omega \tag{1}$$

The continuous partial differential equation (PDE) is replaced with a discrete approximation [1]. So, we have a linear system $Ax = b$. It is easy to solve this coefficient matrix with symmetry. I inspect the differences to approximate and true values. As the order of matrix becomes large, the number of iteration increases. From a numerical examples, we use some preconditioning method [2, 3] to reduce the number of iteration, but we found an error increasing.

References

[1] G. Strang, Differential Equations and Linear Algebra, Wellwsley - Cambridge Press, USA, 2014.

[2] T. Kohno, H. Kotakemori, H. Niki and M. Usui, *Improving modified Gauss-Seidel method for Z-matrices*, Linear Algebra Appl. 267 (1997) 113–123.

[3] T. Kohno, *On the applied precondtionend iterative method*, JSIAM 15, 3 (2005) 235–243.

This study was supported by JSPS KAKENHI Grant Number JP 26400181.

A Second-Order Rational Difference Equation with a Quadratic Term

YEVGENIY KOSTROV

Department of Mathematics and Computer Science, Manhattanville College
Purchase, NY 10577, USA
yevgeniy.kostrov@mville.edu

ZACHARY KUDLAK*

Department of Mathematics, Monmouth University
West Long Branch, NJ 07764, USA
zkudlak@monmouth.edu

Recently, many people have investigated rational difference equations with non-linear terms, see [1, 2, 3] for several examples. We give the boundedness character, local and global stability of solutions of the following second-order rational difference equation with quadratic denominator,

$$x_{n+1} = \frac{\alpha + \gamma x_{n-1}}{Bx_n + Dx_n x_{n-1} + x_{n-1}} \text{ for } n = 0, 1, \dots,$$

where the coefficients are positive numbers, and the initial conditions x_{-1} and x_0 are nonnegative numbers such that the denominator is nonzero. In particular, we show that in a certain region, the unique equilibrium is globally asymptotically stable, while in another region, the equilibrium is a saddle and there exist prime period-two solutions.

References

[1] M. Dehghan, C. M. Kent, R. Mazrooei-Sebdani, N. L. Ortiz and H. Sedaghat, *Monotone and oscillatory solutions of rational difference equations containing quadratic terms*, J. Difference Equ. Appl. 14 (2008) 191–208.

[2] M. DiPippo, E. Janowski and M. R. S. Kulenović, *Global asymptotic stability for quadratic fractional difference equation of order two*, J. Difference Equ. Appl. 17 (2011) 467–477.

[3] S. Kalabušić, M. R. S. Kulenović and M. Mehuljić, *Global period-doubling bifurcation of quadratic fractional second order difference equation*, Discrete Dyn. Nat. Soc. 2014, Art. ID 920410, 13 pp.

[4] Y. Kostrov and Z. Kudlak, *On a second-order rational difference equation with a quadratic term*, Int. J. Difference Equ. (Accepted May 8, 2016).

The adjoint equation method for constructing first integrals of delay differential equations

ROMAN KOZLOV

Department of Business and Management Science
Norwegian School of Economics
Helleveien 30, 5045, Bergen, Norway
Roman.Kozlov@nhh.no

A new method for finding first integrals of delay differential equations is presented. It can be used for delay differential equations which do not possess a variational (Lagrangian) formulation. The method is based on a newly established identity which links symmetries of the underlying delay differential equations, solutions of the adjoint equations and first integrals. The method can be considered as a generalization of the previous research concerning with first integrals of differential and discrete equations. If a sufficient number of first integrals can be obtained, it is possible to find the general solution of the delay differential equations.

References

[1] N. H. Ibragimov, *Nonlinear self–adjointness and conservation laws*, J. Phys A: Math. Gen. 44 (2011) 432002.

[2] V. Dorodnitsyn, E. Kaptsov, R. Kozlov and P. Winternitz, *The adjoint equation method for constructing first integrals of difference equations*, J. Phys. A: Math. Theor. 48 (2015) 055202.

Nonoscillatory solutions of the four-dimensional neutral difference system

JANA KREJČOVÁ

Faculty of Informatics and Statistics, University of Economics
W. Churchill Sq. 4, 130 67 Prague, Czech Republic
jana.krejcova@vse.cz

The purpose of this talk is to present some results of studying the nonlinear neutral difference system

$$\begin{aligned} \Delta\left(x_n + p_n x_{n-\sigma}\right) &= A_n\, f_1\left(y_n\right) \\ \Delta y_n &= B_n\, f_2\left(z_n\right) \\ \Delta z_n &= C_n\, f_3\left(w_n\right) \\ \Delta w_n &= D_n\, f_4\left(x_{\gamma_n}\right), \end{aligned} \tag{S}$$

where $n \in \mathbb{N}_0 = \{n_0, n_0+1, \ldots\}$, n_0 is a positive integer, σ is a nonnegative integer, $\{A_n\}, \{B_n\}, \{C_n\}, \{D_n\}$ are positive real sequences defined for $n \in \mathbb{N}_0$. Δ is the forward difference operator given by $\Delta x_n = x_{n+1} - x_n$. The sequence $\gamma : \mathbb{N} \to \mathbb{N}$ satisfies

$$\lim_{n\to\infty} \gamma_n = \infty.$$

The sequence $\{p_n\}$ is a sequence of the real numbers and it satisfies $0 \le p_n < 1$. Functions $f_i : \mathbb{R} \to \mathbb{R}$ for $i = 1, \ldots, 4$ satisfy

$$\frac{f_i(u)}{u} \ge 1, \quad u \in \mathbb{R}\backslash\{0\}.$$

We study nonoscillatory solutions of (S) and we state asymptotic properties of solutions which lead to establishing sufficient conditions for the system to have weak property B and property B.

References

[1] Z. Došlá and J. Krejčová, *Nonoscillatory solutions of the four-dimensional difference system*, Qualitative Theory of Diff. Equ. 4 (2011) 1–11.

[2] J. Krejčová, *Nonoscillatory solutions of the four-dimensional neutral difference system*, Springer Proceedings in Mathematics & Statistics 164, submitted 2015.

On topological invariants of Brouwer flows

ZBIGNIEW LEŚNIAK

Institute of Mathematics, Pedagogical University
Podchorążych 2, 30-084 Kraków, Poland
zlesniak@up.krakow.pl

We study topological properties of Brouwer flows, i.e. flows of the plane which have no fixed points. For each Brouwer flow we have a decomposition of the plane into parallelizable regions with the transition maps between parallelizing homeomorphisms of these regions. We divide the set of all Brouwer flows into classes by the relation of topological equivalence of flows and for each of these classes we take a common index set for the considered decompositions. This allows us to compare the transition maps of Brouwer flows belonging to the same class. The main result gives a sufficient condition for topologically equivalent Brouwer flows to be topologically conjugate. The condition describes the relations between the transition maps of Brouwer flows contained in the same class. This result generalizes a theorem describing conjugacy classes of Reeb flows.

References

[1] T. Homma and H. Terasaka, *On the structure of the plane translation of Brouwer*, Osaka Math. J. 5 (1953) 233–266.

[2] F. Le Roux, *Classes de conjugaison des flots du plan topologiquement équivalents au flot de Reeb*, C. R. Acad. Sci. Paris 328 (1999) 45–50.

[3] S. Matsumoto, *A characterization of the standard Reeb flow*, Hokkaido Math. J. 42 (2013) 69–80.

Recent advances in global stability of monotone maps: application to population dynamics

RAFAEL LUÍS*

Center for Mathematical Analysis, Geometry and Dynamical Systems
Instituto Superior Técnico, University of Lisbon
Lisbon, Portugal
rafael.luis.madeira@gmail.com

EDUARDO C. BALREIRA, SABER ELAYDI

Department of Mathematics, Trinity University
San Antonio, Texas, USA
ebalreir@trinity.edu; selaydi@trinity.edu

In a previous work [2, 3, 4], the authors established global stability for a certain class of models. Based in these results, we introduce and develop in a forthcoming paper [1], a new notion of normal monotonicity of higher dimensional models defined on Euclidean spaces $\mathbb{R}^k$. Under certain conditions, we determine the global stability of normally monotone maps, i.e., if the map has a unique interior fixed point, then it must be globally asymptotically stable.

In this talk we will present the main results of our work and show the effectiveness of our tools providing detailed proofs of the global stability of two important competition models in population dynamics: the Leslie-Gower model and the Ricker competition model.

References

[1] E. C. Balreira, S. Elaydi and R. Luis, *Geometry and global stability of higher dimensional monotone maps*, preprint.

[2] E. C. Balreira, S. Elaydi and R. Luis, *Local stability implies global stability for the planar Ricker competition model*, Discrete Contin. Dyn. Syst. Ser. B 19 (2014) 323–351.

[3] E. C. Balreira, S. Elaydi and R. Luis, *Global dynamics of triangular maps*, Nonlinear Anal. 104 (2014) 75–83.

[4] R. Luis, *On the global dynamics of periodic triangular maps*, to appear in J. Difference Equ. Appl.

Discrete fractional Cucker–Smale optimal control problem

AGNIESZKA B. MALINOWSKA

Faculty of Computer Science, Bialystok University of Technology
15-351 Białystok, Poland
a.malinowska@pb.edu.p

TATIANA ODZIJEWICZ*

Department of Mathematics and Mathematical Economics
Warsaw School of Economics
02-554 Warsaw, Poland
tatiana.odzijewicz@sgh.waw.pl

In this work, we propose and study discrete fractional Cucker–Smale optimal control problem with Grunwald–Letnikov derivatives defined in the sense of Caputo. Necessary conditions of optimality are given and the numerical treatment to some particular problems is proposed.

References

[1] N. R. O. Bastos, R. A. C. Ferreira and D. F. M. Torres, *Necessary optimality conditions for fractional difference problems of the calculus of variations*, Discrete Contin. Dyn. Syst. 29 (2011) 417–437.

[2] N. R. O. Bastos, R. A. C. Ferreira and D. F. M. Torres, *Discrete-time fractional variational problems*, Signal Processing, vol. 91 (2011) 513–524.

Research supported by Polish founds of National Science Center, granted on the basis of decision DEC-2014/15/B/ST7/05270.

Fractional discrete-time of Hegselmann–Krause's type consensus model

DOROTA MOZYRSKA*

Bialystok University of Technology
15-351 Białystok, Poland
d.mozyrska@pb.edu.pl

MAŁGORZATA WYRWAS

Bialystok University of Technology
15-351 Białystok, Poland
m.wyrwas@pb.edu.pl

There has been an increasing interest in recent years in the analysis of multi–agent systems where agents interact accordingly to some local rules. Speaking about consensus we need to imagine a group of individuals who need to act together as a team or committee. An extensive analysis for the models introduced by Krause in [1] or sometimes referred to as the Hegelsmann–Krause model were given in [2] and [3]. In investigations we use interactions between opinions defined like in Hegselmann–Krause models but with included memory by fractional–order operator on the left side. We use the Grünwald–Letnikov–type difference operator. In the paper we investigate various models for the dynamics of discrete-time fractional order opinions by analytical methods and by computer simulations.

References

[1] U. Krause, *A discrete nonlinear and non-autonomous model of consensus formation*, Communications in difference equations, 227–236, Gordon and Breach, 2000.

[2] R. Hegselmann and U. Krause, *Opinion dynamics and bounden confidence mod- els, analysis, and simulations*, J. Artif. Societies Social Simul. 5 (2002) 227–236.

[3] V. D. Blondel and J. M. Hendrickx and F. N. Tsitsiklis, *On krauses multi-agent consensus model with state-dependent connectivity*, IEEE Transactions on Automatic Control 5 (2009) 2586–2597.

The work was supported by Polish founds of National Science Center, granted on the basis of decision DEC-2014/15/B/ST7/05270.

Exploration of combinatorial emergence in coupled boolean network systems

SHINJI NAKAOKA

Graduate School of Medicine, The University of Tokyo
Hongo 7-3-1, Bunkyo-ku, Tokyo, 133-0033, Japan
shinzy.nakaoka@gmail.com

We consider the following system of difference equations which describes state transition of a set of N binary variables $x_i \in \{0, 1\}$:

$$x_i(t+1) = f_i(x_1(t), x_2(t), \ldots, x_n(t)), \tag{1}$$

where function $f_i : \{0, 1\} \to \{0, 1\}$ represents one of logical functions such as AND, OR, XOR and NOT. Boolean network model (1) proposed by S.A. Kauffman in 1969 [1] was proposed to describe random dynamics of gene regulatory network as crude but useful simplification of real gene regulatory network. Boolean network models have been used to understand regulatory dynamics of transcription factors which activate or suppress expression of various genes [2]. It is known that gene expression pattern changes when distinct cells are co-cultured, which could be typically observed for immune cells. Although several patterns exhibited by a boolean network model such as cycling and steady states qualitatively explain observable real patterns of gene expression dynamics, expected dynamical patterns exhibited by a coupled state of two different cell types have not been fully understood yet. In this talk, we consider a coupled boolean network system to investigate the possibility of combinatorial emergence: an emergent dynamical behavior that is given risen from coupling of two different dynamical systems. Potential biological implications are further discussed based on numerical simulation results.

References

[1] Stuart A. Kauffman, *Homeostasis and differentiation in random genetic control networks*, Nature 224 (1969) 177–178.

[2] Réka Albert and Hans G. Othmer, *The topology of the regulatory interactions predicts the expression pattern of the segment polarity genes in Drosophila melanogaster*, J. Theoret. Biol. 223 (2003) 1–18.

Stability of a logistic equation with multiple delays

YUKIHIKO NAKATA

Department of Mathematics, Shimane University
Matsue, Shimane, 690-8504, Japan
ynakata@riko.shimane-u.ac.jp

We discuss stability of a logistic difference equation with multiple delays. Global stability conditions have been studied in papers e.g. [1, 4, 5]. We review the studies and introduce a new global stability condition obtained in [2]. We then visualize stability conditions in a parameter space performing linearized stability analysis [3]. Examples that delay can stabilize the equilibrium will be presented. A part of this talk is based on a collaboration work with Prof. Emiko Ishiwata and Naoyuki Yatsuda.

References

[1] H. Matsunaga, T. Hara and S. Sakata, *Global attractivity for a logistic equation with piecewise constant argument*, NoDEA Nonlinear Differential Equations Appl. 8 (2001), 45–52

[2] Y. Nakata, *Global asymptotic stability beyond* $3/2$ *type stability for a logistic equation with piecewise constant arguments*, Nonlinear Anal. TMA 73 (2010) 3179–3194.

[3] Y. Nakata, N. Yatsuda and E. Ishiwata, *Visualisation of stability regions for logistic difference equations with multiple delays*, RIMS Kokyuroku 1937 (2015) 32–41.

[4] J. W. H. So and J. S. Yu, *Global stability in a logistic equation with piecewise constant arguments*, Hokkaido Math. J. 24 (1995) 269–286.

[5] K. Uesugi, Y. Muroya and E. Ishiwata, *On the global attractivity for a logistic equation with piecewise constant arguments*, J. Math. Anal. Appl. 294 (2004) 560–580.

Set Invariance for Delay Difference Equations

SORIN OLARU*, MOHAMMED LARABA, SILVIU-IULIAN NICULESCU

Laboratory of Signals and Systems (L2S)
CentraleSupeléc-CNRS-Université Paris-Saclay, France
sorin.olaru@centralesupelec.fr

In set-theory, the notion of positive invariance is used to characterize the properties of dynamical systems. A positive invariant set is defined as a collection of initial conditions generating trajectories in forward time within the same set.

In this talk, set invariance properties for linear time-delay systems are addressed. More precisely, the first goal is to review known necessary and/or sufficient conditions for the existence of invariant sets with respect to dynamical systems described by linear discrete time-delay difference equations (dDDEs) of the form:

$$x(k+1) = \sum_{i=0}^{d} A_i x(k-i) \tag{1}$$

where $x(k) \in \mathbb{R}^n$ is the state vector at the time $k \in \mathbb{Z}_+$, $d \in \mathbb{Z}_+$ is the maximal *fixed* time-delay, the matrices $A_i \in \mathbb{R}^{n \times n}$, for $i \in \mathbb{Z}_{[0,d]}$ and the initial conditions are considered to be given by $x(-i) = x_{-i} \in \mathbb{R}^n$, for $i \in \mathbb{Z}_{[0,d]}$. Secondly, the construction of invariant sets in the original state space, also called $\mathcal{D}$-invariant sets, by exploiting the forward mappings is addressed. The notion of $\mathcal{D}$-invariance is appealing since it provides a region of attraction, which is difficult to obtain for time-delay systems without taking into account the delayed states in some appropriate extended state space model.

In this talk we will discuss recent results related to sufficient conditions for the existence of ellipsoidal $\mathcal{D}$-contractive sets for dDDEs, and necessary and sufficient conditions for the existence of $\mathcal{D}$-invariant sets with respect to linear time-varying dDDE stability. Another interesting point is the clarification of the relationship between convexity (convex hull operation) and $\mathcal{D}$-invariance of linear dDDEs. In short, it is shown that the convex hull of the union of two or more $\mathcal{D}$-invariant sets is not necessarily $\mathcal{D}$-invariant, while the convex hull of a non-convex $\mathcal{D}$-invariant set is $\mathcal{D}$-invariant.

References

[1] M.-T. Laraba et al., *Guide on Set Invariance for Delay Difference Equations*, to appear in Annual Reviews in Control, 2016.

On the Hyers-Ulam stability of a first-order linear difference equation

MASAKAZU ONITSUKA*

Department of Applied Mathematics, Okayama University of Science
Okayama 700-0005, Japan
onitsuka@xmath.ous.ac.jp

TOMOHIRO SHOJI

Department of Applied Mathematics, Okayama University of Science
Okayama 700-0005, Japan

In 1998, C. Alsina and R. Ger [1] investigated the Hyers-Ulam stability of the linear differential equation

$$x' = x, \quad t \in I,$$

where the prime denotes the derivative with respect to t; I is a nonempty open interval of $\mathbb{R}$. To be precise, they proved that if a differentiable function $\phi : I \to \mathbb{R}$ satisfies $|\phi'(t) - \phi(t)| \leq \varepsilon$ for all $t \in I$, then there exists a differentiable function $x : I \to \mathbb{R}$ such that $x'(t) = x(t)$ and $|\phi(t) - x(t)| \leq 3\varepsilon$ for all $t \in I$, where ε is a given non-negative constant. When this fact is satisfied, we say $x' = x$ has the "*Hyers-Ulam stability*". Moreover, we call the constant 3 "*Hyers-Ulam stability constant (HUS constant)*" for the differential equation $x' = x$.

In this talk, we consider the first-order linear difference equation

$$\Delta_h x(t) = x(t), \quad t \in h\mathbb{Z}, \tag{1}$$

where

$$\Delta_h x(t) = \frac{x(t+h) - x(t)}{h}, \quad h > 0;$$

and $h\mathbb{Z} = \{hk|\, k \in \mathbb{Z}\}$. The aim of this talk is to clarify that difference equation (1) has the Hyers-Ulam stability with HUS constant 3.

References

[1] C. Alsina and R. Ger, *On some inequalities and stability results related to the exponential function,* J. Inequal. Appl. 2 (1998) 373–380.

[2] S.-M. Jung, Hyers-Ulam-Rassias stability of functional equations in nonlinear analysis, Springer Optimization and Its Applications 48, Springer, New York, 2011.

The growth of positive solutions of difference equations with applications to delay differential equations

MIHÁLY PITUK

Department of Mathematics, University of Pannonia
H-8200 Veszprém, Hungary
pitukm@almos.uni-pannon.hu

In this talk, we first recall our previous result on the growth rates of positive solutions of a sytem of difference equations [2]. Then we show how this discrete result can be used to determine the growth rates of the positive solutions of a class of delay differential equations.

References

[1] M. Pituk, *A Perron type theorem for functional differential equations*, J. Math. Anal. Appl. 316 (2006) 24–41.

[2] R. Obaya and M. Pituk, *A variant of the Krein-Rutman theorem for Poincaré difference equations*, J. Difference Equ. Appl. 18 (2012) 1751–1762.

Boundedness of Solutions in Almost Linear Volterra Difference Equations Using Fixed Point Theory and Lyapunov Functionals

YOUSSEF RAFFOUL

Department of Mathematics, University of Dayton
Dayton Ohio, USA
yraffoul1@udayton.edu

In this talk we consider the almost linear Volterra difference equation

$$\Delta x(n) = a(n)h(x(n)) + \sum_{k=0}^{n-1} c(n,k)g(x(k)) \tag{1}$$

and obtain conditions under which all solutions are bounded, using Krasnoselskii's fixed point theorem. Also, we will display a Lyapunov functional that yields boundedness of solutions and compare both methods through examples.

References

[1] M. Maroun and Y. Raffoul, *Periodic solutions in nonlinear neutral difference equations with functional delay,* J. Korean Math. Soc. 42 (2005) 255–268.

[2] Y. Raffoul, *Periodicity in general delay non-linear difference equationsu sing fixed point theory*, J. Difference Equ. Appl. 10 (2004) 1229–1242.

Integral stability for dynamic systems on time scales

ANDREJS REINFELDS

Institute of Mathematics and Computer Science, University of Latvia
29 Raina bulv., Rīga LV-1459, Latvia
reinf@latnet.lv

We consider the dynamic system in a Banach space on unbounded above and below time scales:

$$\begin{cases} x^{\Delta} = A(t)x + f(t,x,y), \\ y^{\Delta} = B(t)y + g(t,x,y). \end{cases} \tag{1}$$

This system satisfies the conditions of integral separation with the separation constant ν, the integral contraction with the integral contraction constant μ, nonlinear terms are ε-Lipshitz, and the system has a trivial solution. We prove the theorem of asymptotic phase. Using this result and the centre manifold theorem we reduce the investigation of integral stability of the trivial solution of (1) to investigation of integral stability of the trivial solution of the reduced dynamic system

$$x^{\Delta} = A(t)x + f(t,x,u(t,x)).$$

References

[1] M. Bohner and A. Peterson, Dynamic Equations on Time Scales. An Introduction with Applications, Birkhäuser, Boston, Basel, Berlin, 2001.

[2] A. Reinfelds and L. Sermone, *Stability of impulsive differential systems*, Abstract and Applied Analysis, 2013, DOI:10.1155/2013/253647.

[3] A. Reinfelds and Dz. Steinberga, *Dynamical equivalence of quasilinear dynamic equations on time scales*, Journal of Mathematical Analysis 7 (2016) 115-120.

This work was partially supported by the grant No. 345/2012 of the Latvian Council of Science.

Bounded solution of dynamic system on time scale

ANDREJS REINFELDS[1,2]

[1]Institute of Mathematics and Computer Science, University of Latvia
Raiņa bulvāris 29, Rīga LV-1459, Latvia
reinf@latnet.lv

DZINTRA ŠTEINBERGA[2*]

[2]Department of Mathematics, University of Latvia
Zeļļu iela 25, Rīga LV-1002, Latvia
dzintra.steinberga@gmail.com

We consider the dynamic equation in a Banach space on unbounded above and below time scales $\mathbb{T}$:

$$x^{\Delta} = A(t)x + f(t,x), \tag{1}$$

with rd-continuous, regressive right hand side, nonlinear term satisfy the Lipschitz condition

$$|f(t,x) - f(t,x')| \le \varepsilon(t)|x - x'|,$$

and the estimate

$$|f(t,0)| \le N(t) < +\infty.$$

where $N\colon \mathbb{T} \to \mathbb{R}_+$ and $\varepsilon\colon \mathbb{T} \to \mathbb{R}_+$ are integrable scalar functions. Using Green type mapping [1] we find sufficient condition for the existence of bounded solution and investigate it's properties. We give an example in the case of the nonuniform exponential dichotomy.

References

[1] A. Reinfelds and Dz. Steinberga, *Dynamical equivalence of quasilinear dynamic equations on time scales*, Journal of Mathematical Analysis 7 (2016) 115–120.

[2] M. Bohner and A. Peterson, Dynamic Equations on Time Scale, Birkhäuser, Boston, Basel, Berlin, 2001.

This work was partially supported by the grant Nr. 345/2012 of the Latvian Council of Science

Applications of non-autonomous discrete dynamical systems into nonlinear consensus problems

MANSOOR SABUROV

Department of Computational & Theoretical Sciences
International Islamic University Malaysia
Kuantan, 25200, Pahang, Malaysia
msaburov@gmail.com

Historically, an idea of reaching consensus through repeated averaging was introduced by DeGroot (see [1, 3]) for a structured *time-invariant* and synchronous environment. Since that time, the consensus which is the most ubiquitous phenomenon of multi-agent systems becomes popular in various scientific communities, such as biology, physics, control engineering and social science. Roughly speaking, a trajectory of a row-stochastic matrix presents DeGroot's model of the structured *time-invariant* synchronous environment. In [2], Chatterjee and Seneta considered a generalization of DeGroot's model for the structured *time-varying* synchronous environment. A trajectory of a sequence of row-stochastic matrices (a non-homogeneous Markov chain) presents the Chatterjee-Seneta model of the structured *time-varying* synchronous environment.

In this paper, we shall consider *a nonlinear model of the structured time-varying synchronous environment* which generalizes both DeGroot's and the Chatterjee-Seneta models. Namely, by means of multidimensional stochastic hypermatrices, we present an opinion sharing dynamics of the multi-agent system as a trajectory of non-autonomous polynomial stochastic operators (nonlinear Markov operators). We show that the multi-agent system eventually reaches to a consensus under suitable conditions.

References

[1] R. L. Berger, *A necessary and sufficient condition for reaching a consensus using DeGroot's method*, J. Amer. Stat. Assoc. 76 (1981) 415–418.

[2] S. Chatterjee and E. Seneta, *Towards consensus: some convergence theorems on repeated averaging*, J. Appl. Prob. 14 (1977) 89–97.

[3] M. H. DeGroot, *Reaching a consensus*, J. Amer. Stat. Assoc. 69 (1974) 118–121.

This work was supported by Ministry of Higher Education (MOHE) grant FRGS14-141-0382.

On the Stability of an SIR epidemic discrete model with a delay

KAORI SAITO

Department of Industrial Information, Iwate Prefectural University
1-5-1 Kanan, Miyako, Iwate 027-0039, Japan
saito_k@iwate-pu.ac.jp

A model for the spread of disease-causing is treated with some mathematical epidemic discrete equation with a delay. In this talk, we consider the asymptotic stability of a discrete SIR epidemic discrete model with a delay by applying the luxury Liapunov functional.

References

[1] R. M. Anderson and R. M. May, *Population biology of infectious diseases*, Part 1, Nature 280 (1979) 361-367.

[2] S. Elaydi, An Introduction to Difference Equations, Third Edition, Springer, 2005.

[3] Y. Hamaya and K. Saito, *Global asymptotic stability of a delayed SIR epidemic model with diffusion*, to appear.

[4] H. Inaba, Mathematical Models for Demography and Epidemics, University of Tokyo Press, 2002.

[5] J. D. Murray, Mathematical Biology, Third Edition, Springer, 2002.

[6] Y. Takeuchi and W. Ma, *Stability analysis on a delayed SIR epidemic model with density dependent birth process*, Dynam. Contin. Discrete Impuls. Systems 5 (1999) 171–184.

Converse theorems on globally asymptotic stability of solutions for nonlinear difference equations

SEIJI SAITO

Department of Mathematical Sciences, Doshisha University
Kyotanabe 610-0321, Japan
ssaito@mail.doshisha.ac.jp

In this talk we consider initial value problems

$$x(n+1) = f(n, x(n)),\ x(n_0) = x_0$$

where $k \in \boldsymbol{Z}_+ = \{0, 1, 2, \cdots\}, k \geq 1, n_0 \in \boldsymbol{Z}_+, n \in \boldsymbol{Z}_+, x_0 \in \boldsymbol{R}^k, x \in \boldsymbol{R}^k$ and $f(n, x)$ is continuous in $x \in \boldsymbol{R}^k$ for each $n \in \boldsymbol{Z}_+$ with a unique equilibrium point $x_e = f(n, x_e)$ for $n \in \boldsymbol{Z}_+$. In the same way as in T. Yoshizawa [2] we give converse theorems in case that the equilibrium x_e of the above equation is globally asymptotic stability (see definitions in [1]). And also their applications of the converse theorems are dealt with to perturbed equations.

References

[1] S. Elaydi, An Introduction to Difference Equations, Springer, 2005.

[2] T. Yoshizawa, *Some notes on stability of sets and perturbed systems*, Funkcial. Ekvac. 6 (1964) 1–11.

Equations with five variables give the second Feigenbaum constant

MASANORI SHIRO

Human Informatics Research Institute
National Institute of Advanced Industrial Science and Technology
Central 2, 1-1-1 Umezono, Tsukuba, Ibaraki, 305-8568 Japan
shiro@ni.aist.go.jp

Feigenbaum constant is an universal value between simple chaotic models and fractals [1]. A series expansion for the constant is not known yet. It is just found in almost two thousands digits in Briggs' thesis [2].

An universal function $g(x)$ which is required for calculation of the constant, has following properties [3, 4],

1. $g(x)$ is an even function.
2. $g(\alpha x)/\alpha = g(g(x))$
3. $\alpha = 1/g(1)$
4. $g(0) = 1$

We propose the equations with five variables by linearizing above properties. These equations may be calculated by some types of greedy algorithm, that is a set of difference equations.

References

[1] M. J. Feigenbaum, *Quantitative Universality for a Class of Nonlinear Transformations*, J. Stat. Phys. 19 (1978) 25–52.

[2] K. Briggs, *Feigenbaum Scaling in Discrete Dynamical Systems*, Doctor thesis (1997).

[3] S. H. Strogatz, Nonlinear Dynamics and Chaos: With Applications to Physics, Biology, Chemistry, and Engineering.

[4] M. Campanino et al., *On Feigenbaum's Functional Equation* $g \circ g(\lambda x) + \lambda g(x) = 0$ Topology 21 (1982) 125–129.

[5] K. Karamanos and I. S. Kotsireas, J. Franklin Institute (2005).

Nonoscillation theorems for second-order linear difference equations via the Riccati-type transformation

JITSURO SUGIE

Department of Mathematics, Shimane University
Matsue, Shimane, 690-8504, Japan
jsugie@riko.shimane-u.ac.jp

In this talk, nonoscillation problem is dealt for the second-order linear differentce equation

$$c_n x_{n+1} + c_{n-1} x_{n-1} = b_n x_n,$$

where $\{b_n\}$ and $\{c_n\}$ are positive sequences. For all sufficiently large $n \in \mathbb{N}$, the ratios c_n/c_{n-1} and c_{n-1}/b_n play an important role in the results obtained. To be precise, our nonoscillation criteria are described in terms of the sequence

$$q_n = \frac{c_{n-1}}{b_n}\frac{c_n}{b_{n+1}}\frac{c_n}{c_{n-1}} = \frac{c_n^2}{b_n b_{n+1}}.$$

These criteria are compared with those that have been reported in previous researches by using some specific examples. Figures are attached to facilitate understanding of the concrete examples.

References

[1] R. P. Agarwal, Difference Equations and Inequalities: Theory, Methods, and Applications, 2nd ed., Marcel Dekker, New York, 2000.

[2] M. H. Abu-Risha, *Oscillation of second-order linear difference equations,* Appl. Math. Lett. 13 (2000) 129–135.

[3] S. Elaydi, An Introduction to Difference Equations, 3rd ed., Springer, New York, 2005.

[4] J. W. Hooker, M. K. Kwong and W. T. Patula, *Oscillatory second order linear difference equations and Riccati equations*, SIAM J. Math. Anal. 18 (1987) 54–63.

[5] J. W. Hooker and W. T. Patula, *Riccati type transformations for second-order linear difference equations*, J. Math. Anal. Appl. 82 (1981) 451–462.

[6] M. K. Kwong, J. W. Hooker and W. T. Patula, *Riccati type transformations for second-order linear difference equations*, II, J. Math. Anal. Appl. 107 (1985) 182–196.

An evolutionary game model of families' voluntary provision of public goods

AIKO TANAKA*

Graduate School of Economics and Business Administration, Hokkaido University
Sapporo 060-0809, Japan
atanaka@econ.hokudai.ac.jp

JUN-ICHI ITAYA

Graduate School of Economics and Business Administration, Hokkaido University
Sapporo 060-0809, Japan
itaya@econ.hokudai.ac.jp

We consider a two-stage voluntary provision model where individuals in a family contribute to a pure public good and/or a household public good, and an altruistic parent makes a non-negative income transfer to his or her child. The subgame perfect equilibrium derived in the model is analyzed using two evolutionary dynamics games (i.e., replicator dynamics and best response dynamics). As a result, the equilibria with ex-post transfers and pre-committed transfers coexist, and are unstable in the settings of replicator dynamics as well as best response dynamics, whereas the monomorphic states (i.e., all families undertake either ex-post or pre-committed transfers) are stable. An income redistribution policy does not alter the real allocations in the settings of both evolutionary dynamics games, because the resulting real allocations depend on only the total income of society and not on the distribution of individual income.

References

[1] R. C. Cones, J. Itaya and A. Tanaka, *Private provision of public goods between families*, J. Popul. Econ. 25 (2012) 1451–1480.

[2] D. Fudenberg and D. K. Levine, The Theory of Learning in Games, MIT Press, Cambridge, 1998.

[3] L. Samuelson, Evolutionary Games and Equilibrium Selection, MIT Press, Cambridge, 1997.

Prime period 4 behavior of certain piecewise linear system of difference equations where initial condition are some points in positive x-axis

WIROT TIKJHA*

Faculty of Science and Technology, Pibulsongkram Rajabhat University
Phitsanulok, 65000 Thailand
wirottik@psru.ac.th

EVELINA LAPIERRE

Department of Mathematics, Johnson and Wales University
8 Abbott Park Place, Providence, RI, USA
elapierre@jwu.edu

Recently the authors have been working on a comprehensive project examining the global behavior of a family of systems of piecewise linear difference equations. In this presentation we will share the results from a specific system: $x_{n+1} = |x_n| - y_n - 1$ and $y_{n+1} = x_n + |y_n| - 1$ where initial conditions (x_0, y_0) is such that x_0 is an arbitrary non-negative real value, and y_0 is an arbitrary real value. We will show that every solution of this system is either one of two prime period-3 solutions, or one of two prime period-4 solutions or the unique equilibrium solution. We will also share the sufficient conditions for the prime period-4 solutions.

References

[1] E. A. Grove, E. Lapierre and W. Tikjha, *On the global behavior of* $x_{n+1} = |x_n| - y_n - 1$ *and* $y_{n+1} = x_n + |y_n|$, Cubo. 14 (2012) 125–166.

[2] E. A. Grove and G. Ladas, Periodicities in Nonlinear Difference Equations, Chapman Hall/CRC Press, New York, 2005.

[3] W. Tikjha, E. G. Lapierre and Y. Lenbury, *On the global character of the system of piecewise linear difference equations* $x_{n+1} = |x_n| - y_n - 1$ *and* $y_{n+1} = x_n - |y_n|$, Adv. Difference Equ. 2010, 2010: doi:10.1155/2010/573281, 14pp.

[4] W. Tikjha, S. Jintanasonti and Y. Lenbury, *Periodic behavior of solutions of a certain piecewise linear system of difference equations*, Thai J. Math. 13 (2015) 227–236.

[5] W. Tikjha, E. G. Lapierre and Y. Lenbury, *Periodic solutions of a generalized system of piecewise linear difference equations*, Adv. Difference Equ. 2015, 2015:248, 15pp.

Stability for Monostable Wave Fronts of Delayed Lattice Differential Equations

TZI-SHENG YANG*

Department of Mathematics, Tunghai University
Taichung 40704, Taiwan
tsyang@thu.edu.tw

CHENG-HSIUNG HSU

Department of Mathematics, National Central University
Chung-Li 32001, Taiwan
chhsu@math.ncu.edu.tw

JIAN-JHONG LIN

General Education Center, National Taipei University of Technology
Taipei 10608, Taiwan
jjlin@ntut.edu.tw

This talk is concerned with the stability of traveling wave fronts for delayed monostable lattice differential equations given by

$$\dot{x}_j(t) = G(x_{j-n}(t-\tau_{-n}), \cdots, x_{j-1}(t-\tau_{-1}), x_j(t), x_{j+1}(t-\tau_1), \cdots, x_{j+n}(t-\tau_n)), \quad (1)$$

where $j \in \mathbb{Z}$; the positive constants $\tau_{\pm k}$ for $k = 1, \cdots, n$ are discrete time delays and the spatially independent nonlinearity $G(u_{-n}, \cdots, u_0, \cdots, u_n) : \mathbb{R}^{2n+1} \to \mathbb{R}$ is a C^2 function. we study the stability of a traveling wave front by using comparison principles for the Cauchy problem and initial-boundary value problem of the lattice differential equations, respectively. We show that any solution of the Cauchy problem converges exponentially to a traveling wave front provided the initial perturbation around the traveling wave front is restricted in the weighted space whose asymptotic exponential rate at $-\infty$ (in moving coordinate) is greater than that of the traveling wave front.

References

[1] C.-H. Hsu and S.-S. Lin, *Existence and multiplicity of traveling waves in a lattice dynamical system*, J. Differential Equations 164 (2000) 431–450.

[2] C.-H. Hsu and J.-J. Lin and T.-S. Yang, *Stability for monostable wave fronts of delayed lattice differential equations*, J. Dyn. Diff. Eqns. (2015) DOI: 10.1007/s10884-015-9447-9.

Regular iterations connected with an initial value problem of a nonlinear difference equation

MAREK CEZARY ZDUN*

Institute of Mathematics Pedagogical University of Cracow
Kraków 30-081, Poland
mczdun@up.krakow.pl

HOJJAT FARZADFARD

Department of Mathematics, Shiraz Branch, Islamic Azad University
Shiraz, Islamic Republic of Iran
hojjat.farzadfard@gmail.com

Let f be an increasing homeomorphism of $[0, \infty)$ onto itself with no non-zero fixed point such that $d := f'(0)$ exists and $0 < d < 1$. Let $U : \mathbb{N} \to C[0, \infty)$ satisfy the difference equation $f \circ U(n+1) = U(n) \circ f$ with the initial condition $U(0)(x) = \alpha x$, where $\alpha > 0$. The purpose of this talk is a presentation of the properties of asymptotic solutions of the above equation in a dependence on the parameter α, that is the properties of the limits

$$\underline{f_{\alpha,\infty}}(x) := \liminf_{n\to\infty} f^{-n}(\alpha(f^n(x)), \quad \overline{f_{\alpha,\infty}}(x) := \limsup_{n\to\infty} f^{-n}(\alpha(f^n(x))$$

and

$$f_{\alpha,\infty}(x) := \underline{f_{\alpha,\infty}}(x) = \overline{f_{\alpha,\infty}}(x),$$

if the last equality holds.

Put $N_a := \{\alpha > 0 : \underline{f_{\alpha,\infty}}(a) = \overline{f_{\alpha,\infty}}(a)\}$. We prove that for every (some) $a > 0$ set $\mathbb{R}^+ \setminus N_a$ is at most countable and the map $\alpha \to f_{\alpha,\infty}(a)$ is injective if and only if the Schröder equation $\sigma(f(x)) = d\sigma(x)$ has a regularly varying solution. If $N_x = \mathbb{R}^+$ for an $x > 0$ then the family $\{f_{\alpha,\infty}, t > 0\}$ stands a regular multiplicative iteration group, that is

$$f_{\alpha,\infty} \circ f_{\beta,\infty} = f_{\alpha\beta,\infty}, \quad \alpha, \beta > 0$$

and $f_{\alpha,\infty}$ are differentiable at zero. Moreover, $f_{d,\infty} = f$ and $f'_{\alpha,\infty}(0) = \alpha$ for $\alpha > 0$. If f is of class C^1 and $f'(x) = d + O(x^\mu)$ or f is convex or concave then $N_x = \mathbb{R}^+$. Several others properties of $\underline{f_{\alpha,\infty}}$ and $\overline{f_{\alpha,\infty}}$ will be presented.

References

[1] J. Domsta, *Regularly varying solutions of linear functional equations in a single variable- Application to regular iteration*, Wyd. Uniw. Gdańskiego, 2002.

[2] H. Farzadfard and M. C. Zdun, *On a limit formula for regular iterations*, to appear in J. Math. Anal. Appl.

Self-adjoint extensions of minimal linear relation associated with discrete symplectic system

PETR ZEMÁNEK

Department of Mathematics and Statistics, Masaryk University
Kotlářská 2, CZ-61137 Brno, Czech Republic
zemanekp@math.muni.cz

In this talk we discuss self-adjoint extensions of the minimal linear relation associated with the (time reversed) discrete symplectic system

$$z_k(\lambda) = \mathbb{S}_k(\lambda)\, z_{k+1}(\lambda), \qquad \mathbb{S}_k(\lambda) := \mathcal{S}_k + \lambda \mathcal{V}_k, \tag{S$_\lambda$}$$

where $\lambda \in \mathbb{C}$ is the spectral parameter and the $2n \times 2n$ complex-valued matrices $\mathcal{S}_k$ and $\mathcal{V}_k$ are such that

$$\mathcal{S}_k^* \mathcal{J} \mathcal{S}_k = \mathcal{J}, \;\; \mathcal{V}_k^* \mathcal{J} \mathcal{S}_k \text{ is Hermitian}, \;\; \mathcal{V}_k^* \mathcal{J} \mathcal{V}_k = 0, \;\; \Psi_k := \mathcal{J} \mathcal{S}_k \mathcal{J} \mathcal{V}_k^* \mathcal{J} \geq 0 \tag{1}$$

with the skew-symmetric $2n \times 2n$ matrix $\mathcal{J} := \left(\begin{smallmatrix} 0 & I \\ -I & 0 \end{smallmatrix}\right)$, the superscript $*$ denoting the conjugate transpose, and k belonging to a discrete interval, which is finite or unbounded above. Especially, we emphasize that the matrix $\mathcal{S}_k$ is (conjugate) symplectic according to the first identity in (1). The talk is based on a joint research with S. L. Clark, see [1, 2].

References

[1] S. L. Clark and P. Zemánek, *On discrete symplectic systems: Associated maximal and minimal linear relations and nonhomogeneous problems*, J. Math. Anal. Appl. 421 (2015) 779–805.

[2] P. Zemánek and S. L. Clark, *Characterization of self-adjoint extensions for discrete symplectic systems*, J. Math. Anal. Appl. 440 (2016) 323–350.

Roughness of tempered exponential dichotomies

LINFENG ZHOU

School of Mathematics, Sichuan University
Chengdu, Sichuan 610064, Peoples Republic of China
zlf63math@163.com

Tempered exponential dichotomy describes nonuniform hyperbolicity for linear random dynamical systems. In this talk, we present a result on the roughness of the tempered exponential dichotomy. The result is given in Banach spaces without assuming the invertibility and the integrability condition of the Multiplicative Ergodic Theorem. Moreover, the approach we used gives an explicit estimate for perturbations and an explicit form for the exponent and the bound of the tempered exponential dichotomy. This is a joint work with Kening Lu and Weinian Zhang.

References

[1] Z. Lian and K. Lu, Lyapunov Exponents and Invariant Manifolds for Random Dynamical Systems in a Banach Space, Memories AMS, vol. 967, Amer. Math. Soc., Providence, RI, 2010.

[2] L. F. Zhou, K. Lu and W. N. Zhang, *Roughness of tempered exponential dichotomies for infinite-dimensional random difference equations*, J. Differential Equations 254 (2013) 4024–4046.

Non-commutative symmetries for generalized Davey-Stewartson equations

ALEXANDER ZUEVSKY

Institute of Mathematics, Czech Academy of Sciences
Prague, Czech Republic
zuevsky@yahoo.com

The purpose of this talk is to introduce an analog and reveal the Lie-algebraic structure symmetries [3] of the generalized Davey-Stewartson (GDS) system of equations [1, 2] in a non-commutative setting [6]. In particular, we show that the symmetries in the non-commutative case [4, 6] are related to two copies of the Poisson bracket continual Lie algebra over a non-commutative field [5, 7]. Examples and further development is also provided.

References

[1] Ö. Aykanat and F. Güngör, *The generalized Davey-Stewartson equations, its Kac-Moody-Virasoro symmetry algebra and relation to Davey-Stewartson equations*, J. Math. Phys. 47 (2006) no. 1, 013510, 6 pp.

[2] F. Güngör, *On the Virasoro structure of symmetry algebras of nonlinear partial differential equations*, SIGMA Symmetry Integrability Geom. Methods Appl. 2 (2006) paper 014, 7 pp.

[3] B. Champagne and P. Winternitz, *On the infinite dimensional symmetry group of the Davey-Stewartson equation*, J. Math. Phys. 29 (1988) 1–8.

[4] A. N. Leznov and M. V. Saveliev Group-Theoretical Methods for Integration of Non–Linear Dynamical Systems, Progress in Physics Series 15, Birkhauser–Verlag, Basel, 1992.

[5] M. V. Saveliev, *Integro–differential non–linear equations and continual Lie algebras*, Commun. Math. Phys. 121 (1989) 283–290.

[6] A. Zuevsky, *Non-commutative root space Witt, Ricci flow, and Poisson bracket continual Lie algebras*, SIGMA Symmetry, Integrability, Geom. Methods Appl. 5 (2009) 105.

[7] A. Zuevsky, *Lie-Algebraic symmetries of generalized Davey-Stewartson equations*, J. Physics: Conference Series (JPCS), 532, 012030.

Author Index

Elvan Akın, 13
Ziyad AlSharawi, 14
Aija Anisimova, 15

Stephen Baigent, 16
Ignacio Bajo, 17
Eduardo C. Balreira, 43
Elena Braverman, 18
Inese Bula, 19

Yan-Hsiou Cheng, 20
Jim M. Cushing, 2

Emma D'Aniello, 21
Zuzana Došlá, 22

Saber Elaydi, 3, 43
Jeremias Epperlein, 9, 23

Hiroshi Fujiwara, 29

Ewa Girejko, 24, 25
István Győri, 26

Yoshihiro Hamaya, 27
Ken-ichi Hashimoto, 28
Nobuyuki Higashimori, 29
Ryoji Hiraguchi, 30
Takashi Honda, 31
Cheng-Hsiung Hsu, 32, 33, 61
Kuo-Chih Hung, 34

Ryonfun Im, 28
Yuusuke Iso, 29
Jun-ichi Itaya, 59
Yukiko Iwata, 31
Hideaki Izumi, 35

Yoshitsugu Kabeya, 36
Cónall Kelly, 18
Hikmet Kemaloglu, 37
Toshiyuki Kohno, 38
Hiroshi Kokubu, 4
Ryusuke Kon, 5
Yevgeniy Kostrov, 39
Roman Kozlov, 40
Jana Krejčová, 41
Zachary Kudlak, 39
Takuma Kunieda, 28

Evelina Lapierre, 60
Mohammed Laraba, 48
Zbigniew Leśniak, 42
Jian-Jhong Lin, 32, 61
Eduardo Liz, 6
Rafael Luís, 43

Luís Machado, 24
Agnieszka B. Malinowska, 24, 25, 44
Mauro Marini, 22
Natália Martins, 24
Serena Matucci, 22
Dorota Mozyrska, 45

Shinji Nakaoka, 46
Yukihiko Nakata, 47
Silviu-Iulian Niculescu, 48

Tatiana Odzijewicz, 44
Sorin Olaru, 48
Masakazu Onitsuka, 49
Özkan Öztürk, 13

Mihály Pituk, 50
Christian Pötzsche, 7

Michael A. Radin, 19
Youssef Raffoul, 51
Andrejs Reinfelds, 52, 53
Alexandra Rodkina, 18

Mansoor Saburov, 54
Kaori Saito, 55
Seiji Saito, 56

Ewa Schmeidel, 25
Sebastian J. Schreiber, 8
Masanori Shiro, 57
Tomohiro Shoji, 49
Stefan Siegmund, 9
Petr Stehlík, 9
Dzintra Šteinberga, 53
Jitsuro Sugie, 58

Shin-Ei Takahashi, 35
Aiko Tanaka, 59
Wirot Tikjha, 60
Makoto Tsukada, 35

Shi-Liang Wu, 33
Małgorzata Wyrwas, 45

Tzi-Sheng Yang, 61
Jianshe Yu, 10

Małgorzata Zdanowicz, 25
Marek Cezary Zdun, 62
Petr Zemánek, 63
Weinian Zhang, 11
Linfeng Zhou, 64
Alexander Zuevsky, 65